UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

SUPPLIMENTARY EXAMINATION 2015/16

TITLE O F PAPER: MODERN PHYSICS & WAVE OPTICS

COURSE NUMBER: P231

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SEVEN PAGES INCLUDING THE COVER PAGE

THE LAST PAGE CONTAINS INFORMATION THAT MAY BE USEFUL IN SOME PROBLEMS

SOME SOLUTIONS ARE INCOMPLETE WITHOUT DIAGRAMS THAT ILLUSTRATE HOW THE SOLUTION WAS OBTAINED

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GRANTED BY THE CHIEF INVIGILATOR

QUESTION 1

(a) Discuss what conditions must be met by
   i. the path difference, and **(2 marks)**
   ii. the phase angle **(2 marks)**

for two sinusoidal beams to interfere destructively?

(b) A P231 student performs a Young's double slit experiment using a light of unknown wavelength. He places the screen 2.20 m from the slits, and uses slits separated by 1.54 mm. He observes that the tenth dark fringe occurs at 7.26 mm from the centre of the central maximum.
   i. Determine the wavelength of the light used. **(8 marks)**
   ii. What is the spacing between successive maxima? **(4 marks)**

(c) A soap bubble 250 nm thick is illuminated by white light. The index of refraction of the soap film is 1.36. Note that inside and outside the soap bubble is air.
   i. Which visible colours are most reflected? **(5 marks)**
   ii. Which colours are most transmitted? **(4 marks)**

QUESTION 2

(a) A beam of green light is diffracted by a single slit of width 0.550 mm. The diffraction pattern forms on a wall 2.06 m beyond the slit. The distance between the positions of zero intensity on both sides of the bright central maximum is 4.10 mm. Calculate the wavelength of the laser light? **(5 marks)**

(b) Determine the minimum distance between two point sources that the human eye can distinguish at 26.0 cm, assuming a pupil diameter of 2.06 mm, and a wavelength of 453 nm. **(6 marks)**

(c) A beam of light of intensity I_0 linearly polarized along the z-axis travels along the y-axis towards a polarizing filter, which is in the x-z plane with its axis aligned at an angle of 45.0° with the z-axis. A second polarizing filter is placed in front of the first at an angle of 45.0° with the first.

   i. Find the intensity of the beam emerging from the second filter in terms of the incident beam. **(2 marks)**

   ii. Compare the intensity and polarization of the incident and final beams. **(2 marks)**

(a) What minimum thickness of sodium nitrate will make a quarter-wave plate for helium-neon laser light of wavelength $\lambda = 632.8$ nm. The indices of refraction for the O and the E rays are $n_O = 1.587$ and $n_E = 1.336$, respectively. **(5 marks)**

(d) A grating with 600 grooves/mm is used with a white light source. What is the maximum order number for which it is possible to see the entire visible spectrum? **(5 marks)**

QUESTION 3

(a) The spectrum from a star shows a central maximum at 445 nm. What is the temperature of the star? **(2 marks)**

(b) In a few statements, discuss with justification whether the photoelectric effect supports the wave or particle nature of light. **(5 marks)**

(c) When light from a hydrogen lamp at 486.1 nm is used to illuminate a metal, a stopping potential of 0.587 V reduces the photocurrent to zero.

  (i) What is the maximum speed of the ejected electrons? **(3 marks)**

  (ii) Determine the work function, cut-off frequency and cut-off wavelength for this metal. **(7 marks)**

(d) X-rays having energy of 300 keV undergo Compton scattering from a target. If the scattering rays are detected at 30° relative to the incident rays, find

  i. the Compton shift at this angle, and **(5 marks)**

  ii. the energy of the scattered X-ray. **(3 marks)**

QUESTION 4

(a) Consider a hydrogen-like beryllium ion (Be^{3+}), Z = 4.

i. Use the procedure developed by Bohr for the hydrogen atom to find the expression for the energy levels of the ion, and calculate the energies for the first 4 states, and comment on the energy gaps between successive states. **(10 marks)**

ii. Find the ionization energy of the ion. **(3 marks)**

(b) In interstellar space (space between stars in a galaxy) hydrogen can be excited to very high energy levels.

i. Use the empirical Balmer-Rydberg equation to calculate the wavelength of a highly excited hydrogen atom dropping from the $n = 273$ to the $n = 272$ level. **(3 marks)**

ii. Using the Bohr quantum model of the atom a wavelength $\lambda = 0.926$ m is obtained. In view that n is large state the principle associated with this agreement. **(2 marks)**

(c) Either briefly discuss the principle of laser operation. **(7 marks)**

OR

The quantum numbers of the hydrogen atom. **(7 marks)**

QUESTION 5

(a) Determine the energy released in the following reactions.

i. $^{98}_{44}Ru \rightarrow {}^{4}_{2}He + {}^{94}_{42}Mo$, and **(4 marks)**

ii. $^{144}_{60}Nd \rightarrow {}^{4}_{2}He + {}^{140}_{58}Ce$. **(4 marks)**

iii. State with justification which one would occur on its own. **(2 marks)**

(b) Find the number of nucleons (mass number) and the number of protons for the element denoted X in the nuclear reaction shown next: Explain how you get the said values.
$X \rightarrow {}^{14}_{7}Fe + e^{-} + \bar{\nu}$. **(3 marks)**

(c) A sample from a piece of wood found in cave contains 25.0 g of carbon. The sample shows and activity of 250 decays/minute from carbon 14. The half-life for ^{14}C is 5 730 years.

i. What is decay constant λ for the sample? **(2 marks)**

ii. How many carbon atoms are present in the sample assuming that it is basically all carbon. **(2 marks)**

iii. How many carbon 14 nuclei were present in the original sample when the tree was still alive? **(2 marks)**

iv. What was the initial activity of the sample? **(1 mark)**

v. How long ago did the tree die? **(5 marks)**

SOME INFORMATION THAT MAY BE USEFUL IN SOME PROBLEMS

Avogadro's number $A = 6.02 \times 10^{23}$ particles per mole
Bohr radius $a_0 = 5.291\,772 \times 10^{-11}$ m
Boltzmann's constant, $k_B = 1.3801 \times 10^{-23}$ J/K
Compton wavelength $\lambda_C = \frac{h}{m_e c} = 0.002\,43$ nm
Coulomb constant $k_e = 8.987\,551\,788 \times 10^{23}9$ N.m^2/C^2
Radii of orbit for the hydrogen atom $r_n = n^2\, a_0$
Rydberg constant $R_H = 1.097\,373 \times 10^7$ m^{-1}.
Planck's constant, $h = 6.626\,075 \times 10^{-34}$ Js
$\hbar = 1.054\,572 \times 10^{-34}$ Js
$hc = 1.986\,447 \times 10^{-25}$ Jm
$2\pi hc^2 = 3.741\,859 \times 10^{-15}$ J·m^2·s^{-1}
Speed of light in vacuum, $c = 2.997\,924\,58 \times 10^8$ m/s
Stefan-Boltzmann Constant $\sigma = 335.669\,6 \times 10^{-8}$ W/(m^{2}K^2)
Wien's displacement law $\lambda_{max} = \frac{hc}{4.965 k_B T}$

Nuclear Data

Electron charge, $e = 1.602\,177\,33 \times 10^{-19}$ C
Electron mass, $m_e = 9.109\,389\,7 \times 10^{-31}$ kg = 0.000 548 6 u
Neutron, $m_n = 1.674\,928\,6 \times 10^{-27}$ kg = 1.008 665 u
Proton mass, $m_p = 1.672\,623 \times 10^{-27}$ kg = 1.007 276 u

1 atomic mass unit = 1 amu = 1 u = $1.660\,540\,2 \times 10^{-27}$ kg $\equiv$ 931.494 MeV rest mass energy
1 eV = $1.602\,177\,33 \times 10^{-19}$ J: 1 MeV = $1.602\,177\,33 \times 10^{-13}$ J
$T_{1/2}(^{14}C)$ = 5730 years
$T_{1/2}(^{238}U) = 4.47 \times 10^9$ years
Ratio of carbon 14 to carbon 12 in the atmosphere = $\frac{N(^{14}C)}{N(^{12}C)} = 1.2987 \times 10^{-12}$
1Curie (Ci) = 3.7×10^{10} Becquerel (Bq)
$r_0 = 1.2 \times 10^{-15}$ m
Alpha particle (α) (^{4}He) atomic mass = 4.002 603 u
Cerium ($^{140}_{58}$Ce) atomic mass = 139.905 434 u
Deuterium (^{2}D) atomic mass = 2.014 102 u
Hydrogen (^{1}H) atomic mass = 1.007 825 u
Hydrogen molecular mass = 1.007 94 u
Iron (^{56}Fe) atomic mass = 55.934 942 u
Molybdenum (^{94}Mo) mass = 93.905 088 u
Neodymium (^{144}Nd) atomic mass = 143.910 083 u
Oxygen ^{15}O atomic mass = 15 .003 065 u
Nitrogen ^{15}N atomic mass = 15.000 109 u
Radium ($^{226}_{88}Ra$) atomic mass = 226.025 403 u
Radon ($^{222}_{86}Rn$) atomic mass = 222.017 570 u
Ruthenium (^{98}Ru) atomic mass = 97.905 287 u
Tritium (^{3}T) atomic mass = 3.016 049 u

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$