UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2015/2016

TITLE OF THE PAPER: COMPUTATIONAL METHODS-I

COURSE NUMBER: P262

TIME ALLOWED:
SECTION A: ONE HOUR
SECTION B: TWO HOURS

INSTRUCTIONS:
THERE ARE TWO SECTIONS IN THIS PAPER:

- **SECTION A:** IS A WRITTEN PART. ANSWER THIS SECTION ON THE ANSWER BOOK. IT CARRIES A TOTAL OF **40** MARKS.
- **SECTION B:** IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. IT CARRIES A TOTAL OF **60** MARKS.

Answer **all** the questions from Section A and **all the questions** from Section B.

Marks for different sections of each Question are shown in the right hand margin.

THE PAPER HAS 5 PAGES, INCLUDING THIS PAGE.

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Section A – Use a pen and paper to answer these questions

Question 1

(a) Translate the following expressions into Maple input statements

(i) $\frac{\partial}{\partial x} e^{-(x^2+y^2)}$

(ii) $\int_0^{\pi} cos^3(x)$

(iii) $1 + 1/20 + 1/40 + 1/60 + 1/80 + 1/100 + ... + 1/1000$

(iv) $\sqrt{y}e^{-x^2}$

[8 marks]

(b) Determine the output of the following Maple statements:

(i) > $z := 2 + 5 * I$: evalc(z*conjugate(z));

(ii) > $f := x - 2 * x$^2 $- 3 * x + 14 : f(0)$;

(iii) > $y := exp(-x$^2): diff(y,x\$2);

(iv) > $d := (x, y, z) \rightarrow sqrt(x$^2 $+ y$^2 $+ z^2) : d(-1, 2, 5)$;

(v) >mag := proc(a,b)
> a.b;
> end;
> v1:=< 1, 2, 3 >;
> v2:=< 4, 5, 6 >;
> mag(v1,v2);

[5 marks]

(c) Write a Maple procedure that uses the Newton's method for finding the square root of a given number r. The iteration is

$$x_0 = 1; \quad x_{n+1} = \frac{x_n^2 + r}{2x_n}, \quad n = 0, 1, 2....$$

The stopping criterion is when $|x_{n+1} - x_n|$ is small enough - say less than 0.000001.

[7 marks]

Question 2

(a) The Hamiltonian matrix for a physical system is given as

$$\mathbf{A} = \begin{bmatrix} -2r & 0 & 0 & v \\ 0 & 0 & v & 0 \\ 0 & v & 0 & 0 \\ v & 0 & 0 & 2r \end{bmatrix}$$

Write Maple commands to

(i) define **A**

(ii) compute the determinant and the trace of **A**.

[4 marks]

(b) The radioactive decay equation can be solve numerical using the equation

$$N_{i+1} = N_i - \Delta t \frac{N_i}{\tau}$$

where N_i is the number of nuclei present at an instant i, $\tau = 1s$ is time constant. Assume that the time step $\Delta t = 0.1$s and that the initial number of radioactive nuclei is $N_0 = 1000$. Use above algorithm to create a plot of the data points $[t_i, N(t_i)]$ for $t_i = 0..10$s, with $t_i = i \cdot \Delta t$

[6 marks]

(c) A force $\vec{F} = [4\hat{x} - 2y(t)\hat{y}]$ acts on a body of mass $m = 2$. Assume that the initial velocity $\vec{v}(0) = 2\hat{x}$m/s, where $\hat{x}$ and $\hat{y}$ are the unit vectors along the horizontal and vertical axis. We can find the displacement $\mathbf{r}(t) = x(t)\hat{x} + y(t)\hat{x}$ of the body at time t using Newton's second law

$$\frac{d^2x(t)}{dt^2} = F_x/m, \quad \frac{d^2y(t)}{dt^2} = F_y/m$$

where F_x and F_y are the x and y components of the force $\vec{F}$.

(i) Decompose the above equations into a system of four first-order equations.

[4 marks]

(ii) Write a short Maple program to compute the exact solution of the displacement of the body given that the initial position $\vec{r}(t = 0) = [x(0), y(0)] = 0$.

[6 marks]

Section B – Practical Part

Question 3

Balls falling out the sky:- Golf and baseball players claim that a hit balls appear to fall straight down out the sky at the end of their trajectories. Your problem is to determine whether there is a simple physics explanation for this effect or whether it is "all in the mind's eye ."

The basic physics is Newton's second law in two dimensions for a realistic projectile. You want to determine if the inclusion of air resistance leads to trajectories that are much steeper at their ends than at their beginnings. The dynamics of a projectile in the presence of a frictional force opposing motion :

$$m\frac{d^2x(t)}{dt^2} = F_x^f, \quad m\frac{d^2y(t)}{dt^2} = -mg + F_y^f$$

where the frictional force $\mathbf{F^f} = (F_x^f, F_y^f)$ and the acceleration due to gravity $g = 9.8$ m/s^2. Assume that the frictional force is proportional to square of the projectile's speed.

$$\mathbf{F}^f = -km|v|^2\frac{\mathbf{v}}{|v|}$$

where the $-\mathbf{v}/|v|$ factor ensures that the frictional force is always in a direction opposite that of the velocity. With a constant power law for friction , the equations of motions are

$$\frac{d^2x(t)}{dt^2} = -k|v|v_x(t), \quad \frac{d^2(t)}{dt^2} = -g - k|v|v_y(t), \quad |v| = \sqrt{v_x(t)^2 + v_y(t)^2},$$

where $v_x(t) = dx(t)/dt$ and $v_y(t) = dy(t)/dt$. Your may assume that the ball is launched with $\mathbf{v}(t=0) = v_0[\cos(\theta), \sin(\theta)]$, with $v_0 = 120$m/s and $\theta = 15$ degrees. Adjust the values of k to find the conditions where the hit ball falls from the sky. Explain the behavior of the projectile for the different values of k.

[30 marks]

Question 4

Debye's theory of solids gives the heat capacity of a solid at temperature T to be

$$C_V = 9V\rho k_B\left(\frac{T}{\theta}\right)^3\int_0^{\theta/T}\frac{x^4e^x}{(e^x-1)^2}dx,$$

where V is the volume of the solid, ρ is the number density of atoms, $k_B = 1.38 \times 10^{-23}\text{m}^2\text{kg}\,\text{s}^{-2}\text{K}^{-1}$ is Boltzmann's constant, and θ is the so-called *Debye temperature*, a property of solids that depends on their density and speed of sound.

(a) Write a Maple procedure *cv(T)* that calculates C_V for a given value of the temperature, for a sample consisting of 1000 cubic centimeters of solid aluminum, which has a number density of $\rho = 6.022 \times 10^{28}$ m^{-3} and a Debye temperature of $\theta = 428$ K.

(b) Use your function to make a graph of the heat capacity as a function of temperature from T = 5K to T = 500K.

[15 marks]

Question 5

The electrostatic potential $V(x)$ inside a cylinder was measured and the following results were obtained :

$$\begin{aligned} V(0.0) &= 52.640,\ V(0.2) = 48.292,\ V(0.4) = 38.270, \\ V(0.6) &= 25.844,\ V(0.8) = 12.648,\ \mathit{and}\, V(1.0) = 0.0, \end{aligned} \tag{1}$$

where x is the distance from the center of the cylinder in metres and V is given in Volts.

(a) Plot the a graph of $V(x)$ versus x using the given data set.

[5 marks]

Symmetry arguments requires that $V(x)$ be an even function of x. Given that

$$V(x) = \sum_{n=0}^{5} a_{2n}x^{2n} = a_0 + a_2x^2 + a_4x^4 + a_6x^6 + a_8x^8 + a_{10}x^{10},$$

(b) Use the LeastSquare procedure in Maple to estimate the values of a_{2n} from the information given in Eq. (1).

[5 marks]

(c) At what distance between $x = 0.0$ and $x = 1.0\,m$ is the electric potential equal to 20 Volts? *You may need to find the roots of the equation:* $V(x) - 20 = 0$.

[5 marks]