UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2015/2016

TITLE OF PAPER: NUCLEAR PHYSICS

COURSE NUMBER: P442

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE AND THE LAST PAGE WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

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Useful Data:

1 unified mass unit $(u) = 1.6605 \times 10^{-27}$ kg $= 931.5$ MeV$/c^2$

Planck's constant $h = 6.63 \times 10^{-34}$ Js

Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K

Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1}

Speed of light (vacuum) $c = 3.0 \times 10^8$ m/s

electron mass $m_e = 9.11 \times 10^{-31}$ kg $= 5.4858 \times 10^{-4}$ u $= 0.511$ MeV$/c^2$

neutron mass $m_n = 1.6749 \times 10^{-27}$ kg $= 1.008665$ u $= 939.573$ MeV$/c^2$

proton mass $m_p = 1.6726 \times 10^{-27}$ kg $= 1.0072765$ u $= 938.280$ MeV$/c^2$

$1 year = 3.156 \times 10^7$ s

nuclear radius, $R \approx r_0 A^{1/3}$, where $r_0 = 1.2$ fm

fine structure constant, $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}$

$\hbar c = 197$ MeVfm

The table of nuclear properties is provided in the next page.

Nuclide	Z	A	Atomic mass (u)	I^π	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	0^+	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8 s (β^-)
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.00000	0^+	99.89%
N	7	15	15.00109	$1/2^-$	0.366%
N	7	18	18.014081	1^-	0.63 s
O	8	15	15.003065	$1/2^-$	122 s
O	8	16	15.994915	0^+	99.76%
O	8	18	17.999160	0^+	0.204%
F	9	18	18.000937	1^+	110.0 min
Ne	10	20	19.992436	0^+	90.51%
Ne	10	22	21.991383	0^+	9.33%
Na	11	22	21.994434	3^+	2.60 yrs
Mg	12	21	21.000574	0^+	3.86 s
Al	13	27	26.981539	$5/2^+$	100.0%
Si	14	30	29.973770	0^+	3.10%
Si	14	32	31.974148	0^+	105 yrs
P	15	30	29.978307	1^+	2.50 min
P	15	32	31.971725	1^+	14.3 days
S	16	32	31.972071	0^+	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 days
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 days (β^-)
Sc	21	47	46.952409	$7/2^-$	3.35 days (β^-)
Fe	26	56	55.934439	0^+	91.8%
Fe	26	60	59.934078	0^+	1.5 Myrs
Co	27	60	59.933820	5^+	5.27 yrs
Ni	28	60	59.930788	0^+	26.1%
Ni	28	64	63.927968	0^+	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 hrs (β^-)
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	1^+	12.7 hrs
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	0^+	48.6%
Ru	44	104	103.905424	0^+	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44 hrs (β^-)
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 yrs (β^-)
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 days (β^-)
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%

Question 1

(a) Consider the scattering of 10 MeV α particles by a gold nucleus (Z=79,A=197). Recall that the Rutherford formula for the differential cross section is

$$\frac{d\sigma}{d\Omega}=\left(\frac{zZe^2}{16\pi\epsilon_0 T}\right)^2\frac{1}{\sin^4(\theta/2)},$$

where T is the kinetic energy

i. Using the fine structure constant and other constants, show that the Rutherford formula can be written as

$$\frac{d\sigma}{d\Omega}=\frac{z^2Z^2}{16}\left(\frac{197}{137T(MeV)}\right)^2\frac{1}{\sin^4(\theta/2)}fm^2/sr,$$

where sr (Steradian) is the SI unit of the solid angle.

ii. Calculate the differential cross section for scattering at 10°; 20° and 30°.

iii. Describe the anomaly that happens to the differential cross as the angle decreases? What assumption(s) causes this scenario? How can the assumption(s) be relaxed to correct the anomaly?

(b) Calculate the potential energy in MeV due to Coulomb repulsion of

i. two protons separated by a distance of 1 fm;

ii. a gold nucleus and an α-particle with their centers at 10 fm apart;

iii. two nuclei $Z = 46$, $A = 115$, which are **just** not touching.

Question 2

(a) The mean life (τ) for for the K^+-meson is 1.237×10^{-8} s. The table below shows the different decay modes of the meson together with the different branching ratios. Copy the table and fill in the last column, i.e calculate the partial transition rates. (1

Decay Mode	Branching fraction	Partial transition rate (s^{-1})
$K^+ \to \mu^+ + \nu_\mu$	0.635	
$K^+ \to \pi^+ + \pi^0$	0.212	
$K^+ \to \pi^+ + \pi^+ + \pi^-$	0.056	
$K^+ \to \pi^+ + \pi^0 + \pi^0$	0.017	
$K^+ \to \pi^0 + \mu^+ + \nu_\mu$	0.032	
$K^+ \to \pi^0 + e^+ + \nu_e$	0.048	

(b) A sample of gold is exposed to a neutron beam of constant intensity such that 10^{10} neutrons per second are absorbed in the reaction

$$^{197}_{79}\mathrm{Au} + n \to ^{198}_{79}\mathrm{Au} + \gamma.$$

The nuclide $^{198}_{79}$Au undergoes β-decay to $^{198}_{80}$Hg with a mean life of 3.89 days.

i. How many seconds are in 3.89 days and in 6 days? (

ii. How many atoms of $^{198}_{79}$Au will be present after 6 days of irradiation?

iii. How many atoms of $^{198}_{80}$Hg will be present at that time, assuming that this nuclide is not affected by the neutron beam?

iv. What is the equilibrium number of $^{198}_{79}$Au atoms?

Question 3

(a) For the following γ transitions, give all permitted multipoles and indicate which multipole might be most intense in the emitted radiation.

i. $\frac{9}{2}^- \rightarrow \frac{7}{2}^+$ (3

ii. $\frac{1}{2}^- \rightarrow \frac{7}{2}^-$ (3

iii. $1^- \rightarrow 2^+$ (3

iv. $4^- \rightarrow 2^+$ (3

v. $\frac{11}{2}^- \rightarrow \frac{3}{2}^+$ (3

(b) Explain why a transition from 0^+ to 0^+ will not allow any γ radiation. (2

(c) An Even-Z, even-N nucleus has the following sequence of levels above its 0^+ ground state: (8
2^+(89 keV), 4^+(288 keV), 6^+(585 keV), 0^+(1050 keV), 2^+(1129 keV).
Draw an energy level diagram and show all reasonably probable γ transitions and their dominant multipole assignments.

Question 4

(a) The Q value can be used to determine if a nuclide is energetically stable against alpha decay or not. Use nuclear masses provided in the table found in the information pages to answer some of the following questions.

i. Define the Q value of alpha emission in terms of nuclear masses. (

ii. What values of Q indicate a nuclide is energetically unstable against alpha emission? (

iii. Calculate Q values for alpha emission by the following nuclides: $^{203}_{81}\mathrm{Tl}$, $^{64}_{30}\mathrm{Zn}$, $^{22}_{11}\mathrm{Na}$ and $^{11}_{5}\mathrm{B}$. **Note**: The resulting daughter nuclide for alpha emission is in the information table. (1

iv. Comment on the relation between stability against alpha emission and the mass number A. (

(b) Lets consider a nucleus at rest which then undergoes alpha emission. We define the disintegration energy E as the sum of the kinetic energy of the alpha particle E_α (mass m_α) and kinetic energy of the recoiling daughter nucleus (mass m_d). Use conservation of linear momentum to show that (

$$E = E_\alpha \left(1 + \frac{m_\alpha}{m_d}\right).$$

Question 5

(a) In classical electrodynamics, the scalar field $\phi(\vec{r})$ produced by an electron located at the origin is given by the Poisson equation (1(

$$\nabla^2\phi(\vec{r}) = -4\pi\delta(\vec{r}).$$

Show that the field is given by $\phi(r) = \frac{e}{r}$.

(b) For a nucleon, the scalar field satisfies the Klein-Gordon equation (1(

$$\left(\nabla^2 - \frac{1}{r_0^2}\right)\phi(\vec{r}) = 4\pi g\delta(\vec{r}),$$

where r_0 is the effective range of the interaction described by the field. Show that the radial dependence of the field is given by

$$\phi(r) = -g\frac{e^{-r/r_0}}{r}.$$

(c) Show that the range r_0, in the above equation is given by the relation $r_0 = \hbar/mc$ using the fact that the boson, with mass m, is a virtual particle and can therefore exist only for a time Δt given by the Heisenberg uncertainty relation. (!

NB In spherical coordinates

$$\delta(\vec{r}) \equiv \frac{1}{r^2}\delta(r)\delta(\cos\theta)\delta(\varphi);$$

$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi}$$

IDENTITIES

$$\nabla^2 u \equiv \nabla\cdot\nabla u$$

$$\int dx\frac{1}{x_0}\exp(x/x_0) = \ln(x/x_0) + \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}(x/x_0)^n$$

Gauss' Divergence theorem

$$\int_V d\vec{r}\nabla\cdot\nabla u(r) = \int_S dS\frac{du(r)}{dr}$$