UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2015/2016

TITLE OF THE PAPER: COMPUTATIONAL METHODS-II

COURSE NUMBER: P482

TIME ALLOWED:
SECTION A: ONE HOUR
SECTION B: TWO HOURS

INSTRUCTIONS:
THE ARE TWO SECTIONS IN THIS PAPER:

- **SECTION A** IS A WRITTEN PART. ANSWER THIS SECTION ON THE ANSWER BOOK. IT CARRIES A TOTAL OF **40** MARKS.
- **SECTION B** IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. IT CARRIES A TOTAL OF **60** MARKS.

Answer **all** the questions from **section A** and
all the questions from **section B.**
Marks for different sections of each Question are shown in the right hand margin.

THE PAPER HAS 5 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

Section A

Question 1

(a) Convert the following into a valid F95 or C++ code

(i) $\frac{a-b}{c+4d}$

(ii) $\sqrt{|ab|}$

(iii) $(e^{2a})^3$

(iv) $\frac{a}{bc}$

(v) $\tan^{-1}\sqrt{\sin^3 x}$

[5 marks]

(b) Trace the following program and predict its output. Assume an input of 1, 1, 1, 2, 3, and 4, respectively.

```
IMPLICIT NONE
REAL*8 :: x1,x2,y1,y2,z1,z2, ans
WRITE (*,*) ' enter two 3 component vectors'
READ *, x1, y1, z1, x2, y2, z2
ANS =0
ANS = x1*x2 + ANS
ANS = y1*y2 + ANS
ANS = z1*z2 + ANS
WRITE(*,*) ' The answer is', ANS
END
```

[3 marks]

(c) Locate errors in the following loop J =1

```
DO WHILE (J<=100)
PRINT *, J
J = J-1
END DO
```

[3 marks]

Question 2

The quantum simple harmonic oscillator has energy levels $E_i = \hbar\omega(i + \frac{1}{2})$, where i = 0, 1, 2, 3, ...,N. The average of energy of the system is

$$\langle E \rangle = \frac{1}{Z} \sum_{i=0}^{N} E_i \exp(-\beta E_i)$$

where β is the inverse temperature and $Z = \sum_{i=0}^{N} \exp(-\beta E_i)$. Suppose we want to calculate, approximately the value of $\langle E \rangle$ when $\beta = 0.01$. Write a F95/C++ function that calculates the of $\langle E \rangle$ with an input N, working in units where $\hbar = \omega = 1$.

[10 marks]

Question 3

Use the Euler method to solve

$$\frac{dN(t)}{dt} = -N(t)$$

with $N(0) = 1$.

(a) Determine $N(t_i)$ after i steps of size Δt. What is the exact solution?

[5 marks]

(b) For $\Delta t = 0.5, 1.5, 2$ calculate $N(t_i)$ for $i = 0...4$. Sketch $N(t_i)$ vs t_i. Which values of Δt is the method stable.

[5 marks]

Question 4

(a) One of the most famous series is that due to Fibonacci:

1, 1 2, 3, 5, 8, 13, 21, 34, ...

This series is known to describe population explosion among rabbits. The first two numbers in the series are 1 and 1. All the additional terms of the series are the sum of the previous terms. Write a program to calculate the first N terms of the series.

[7 marks]

(b) Compute the period of the sequence (x_n) defined below. Give two reasons why it would make a bad random number generator.

$$\begin{aligned} x_n &= (x_{n-1} + x_{n-2}) \bmod 7 \\ x_0 &= 3 \\ x_1 &= 5 \end{aligned}$$

[3 marks]

Section B

***Note*: The answers to this question must include the computer code and output, in addition to any writing that might be needed.**

Question 3

The dynamics of a charged particle in a magnetic field is described by Newton's second law:

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}\mathbf{v} \times \mathbf{B} - \frac{\gamma}{m}\mathbf{v}$$

where $\mathbf{B}$ is the magnetic field, γ represent the coefficient of a damping force, m and q corresponds to the mass and the charge of the particle, respectively.

(a) **Uniform field.** When $\mathbf{B}$ is in the z-direction, then motion is described given by four equations:

$$\begin{aligned} \frac{dx(t)}{dt} &= v_x(t) \\ \frac{dy(t)}{dt} &= v_y(t) \\ \frac{dv_x(t)}{dt} &= \frac{qB}{m}v_y(t) - \frac{\gamma}{m}v_x(t) \\ \frac{dv_y(t)}{dt} &= -\frac{qB}{m}v_x(t) - \frac{\gamma}{m}v_y(t) \end{aligned}$$

(i) Write a $F95$ program to simulate the dynamics of the charged particle using the Euler algorithm. Assume that the initial velocity of the particle is $\mathbf{v}(t=0) = (1,0)$ and the initial position $\mathbf{r}(t=0) = (0,0)$. Let $\Delta t = 0.001$, $qB/m = 2.0$, and $\gamma/m = 0$ and the number of iteration $N_t = 10000$. All this quantities are given in dimensionless units. Plot the trajectory of the particle **r**. Plot the trajectory of the particle, .i.e, x(t) vs y(t). What is the shape of the trajectory?

[30 marks]

(ii) Choose an appropriate value(s) of γ/m to investigate the effects of the damping force. Plot the trajectory of the particle under the influence of a damping force with the same initial conditions as (i). Discuss your observations.

[10 marks]

(b) **Crossed Electric-Magnetic fields.** Now suppose an electric field $\mathbf{E} = E_0\hat{\mathbf{x}}$ is introduced into the system . In this case we need to add an extra term in the equation describing the change in x-component of the velocity :

$$\frac{dv_x(t)}{dt} = \frac{qB}{m}v_y(t) - \frac{\gamma}{m}v_x(t) + \frac{qE}{m},$$

whilst the other equations remain the same. Assume that $qE/m = 2.0$, $qB/m = 2.0$, and $\gamma/m = 0$, in dimensionless units. Show that the particle released with an initial velocity $\mathbf{v}(t) = 0$ from the origin will move like a leaping frog along the y-axis. Explain why this is so.

[20 marks]