UNIVERSITY OF SWAZILAND

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FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2016/2017

TITLE OF PAPER: SOLID STATE PHYSICS

COURSE NUMBER: P 412

TIME ALLOWED : THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

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Question One

(a) Sate two properties of a ***primitive*** unit cell. (2 marks)

(b) Draw ***one*** figure showing a conventional unit cell, and also its primitive cell, for an f.c.c lattice.
(4 marks)

(c) One side of a conventional unit cell of an f.c.c. lattice is 3 Å. What is the volume of its primitive unit cell?
(2 marks)

(d) Calculate the separation between two (123) planes of an orthorhombic lattice with cell lengths, a = 0.82 nm, b = 0.94 nm and c = 0.75 nm in the x, y, z directions.
(3 marks)

(e) Compute the packing fraction of a b.c.c lattice. (4 marks)

(f) Find the indices of the (100) planes of an f.c.c lattice as referred to its primitive axes.
(4 marks)

(g) Show that the reciprocal lattice of a b.c.c is f.c.c. (6 marks)

Given:

Primitive translation vectors of fcc are:

$$a_1 = \frac{1}{2}a(\hat{x} + \hat{y}), b_1 = \frac{1}{2}a(\hat{y} + \hat{z}), c_1 = \frac{1}{2}a(\hat{z} + \hat{x})$$

Primitive translation vectors of bcc are:

$$a_1 = \frac{1}{2}a(\hat{x} + \hat{y} - \hat{z}), b_1 = \frac{1}{2}a(-\hat{x} + \hat{y} + \hat{z}), c_1 = \frac{1}{2}a(\hat{x} - \hat{y} + \hat{z})$$

Question Two

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(a) (i) Derive the Bragg law $2d \sin\theta = n\lambda$ for diffraction of waves by a crystal lattice. (5 marks)

(ii) In the X-ray photograph of a cubic lattice, lines are observed at the following Bragg angles (in degrees): 6. 6, 9. 2, 11.4 ,13.1, 14.7, 16.1, 18.6, 19.8. Assign Miller indices to these lines and identify the lattice type. (5 marks)

(b) (i) A wave of wave vector **k** is incident on a crystal specimen. The diffracted wave has wave vector **k′**. Show that the diffraction condition for constructive interference between the two waves can be written as:

G = **Δk**, where **Δk** = **k** - **k′** and **G** is a reciprocal lattice vector.

What is the physical meaning of the above diffraction condition?

$$[\text{Given: } n(\bar{r}) = \sum_G n_G \exp i\bar{G}.\bar{r}\ ]$$

(10 marks)

(ii) Show that the above diffraction condition **G** = **Δk** is equivalent to the Bragg law $2d \sin\theta = n\lambda$ (5 marks)

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Question Three

(a) By considering the boundary conditions on lattice waves in a cubic crystal, show that for a three dimensional system, the density of states of elastic waves of frequency ω

$D(\omega) = \dfrac{V\omega^2}{2\pi^2 v^3}$ where V is the volume and v the velocity of sound.

(6 marks)

(b) (i) What are the assumptions in the *Debye model* of the theory of specific heat?

(3 marks)

(ii) Show that by applying the above assumptions, the density of states for a system with one atom per unit cell is given by

$D(\omega) = \dfrac{9N\omega^2}{\omega_D^{\ 3}}$ where N is the number of atoms and ω_D is the Debye frequency.

(6 marks)

(iii) Use the above results to show that at low temperature T, the specific heat of an insulator $C_V \propto T^3$. (10 marks)

Given: The mean energy of a harmonic oscillator $\bar{\varepsilon} = \hbar\omega\left(\dfrac{1}{2} + \dfrac{1}{e^{\hbar\omega/kT} - 1}\right)$

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Question Four

(a) State the assumptions Drude made in his *free electron theory* of metals. (3 marks)

(b) Define the terms *mean free path* and *mobility* of an electron. (2 + 2 marks)

(c) Show that according to Drude theory, electrical conductivity of a metal can be expressed as

$\sigma = ne\mu$ where the symbols have their usual meanings. (10 marks)

(d) (i) State the *Wiedemann - Franz* law. (2 marks)

(ii) Write down the expression for *Lorenz number L* and calculate its value. (2 + 2 marks)

(iii) Experimentally the value of *L* at low temperatures is slightly less than its value at high temperatures. Suggest a possible reason for this observation. (2 marks)

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Question Five

(a) State what is meant by *effective density of states* of a semiconductor (2 marks)

(b) With the help of appropriate energy diagram, show that the effective density of states of electrons in the conduction band of a semiconductor.

$$N_c = 2\left(\frac{2\pi mkT}{h^2}\right)^{3/2}$$

[Assume: $(\epsilon - \epsilon_F) \gg kT$]

Given: Fermi -Dirac distribution function: $f(\varepsilon) = \dfrac{1}{e^{(\varepsilon - \varepsilon_F)/kT} + 1}$

Density of states for a system of fermions $D(\varepsilon)d\varepsilon = \dfrac{4\pi}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$

(12 marks)

(c) Calculate:

(i) the effective density of states in the conduction and valance bands of silicon (2 + 2 marks)

(ii) the intrinsic carrier concentration of silicon. (3 marks)

[Effective masses of electrons and holes are 1.1 m_0 and 0.56 m_0, respectively. Band gap of silicon = 1.1 eV]

(d) A doped semiconductor has electron and hole concentrations 2×10^{13} cm^{-3} and 1.41×10^{13} cm^{-3} respectively. Calculate the electrical conductivity of the sample. (4 marks)

[Take: $\mu_n = 4200$ $cm^2\ V^{-1}s^{-1}$. $\mu p = 2000$ $cm^2\ V^{-1}s^{-1}$]

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Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

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Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} Js
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan- Boltzmann constant	σ	5.67×10^{-8} Wm^{-2} K^{-4}
Atmospheric pressure		1.01 10^{5} Nm^{-2}
Mass of $_2^4$ He atom		6.65×10^{-27} kg
Mass of $_2^3$ He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 L mol^{-1}