UNIVERSITY OF SWAZILAND

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FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2016/2017

TITLE OF PAPER: NUCLEAR PHYSICS

COURSE NUMBER: P442

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT TWO PAGES WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

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Useful Data:

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1 unified mass unit (u) = 1.6605×10^{-27} kg = 931.5 MeV/c^2

Planck's constant $h = 6.63 \times 10^{-34}$ Js

Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K

Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1}

Speed of light (vacuum) $c = 3.0 \times 10^8$ m/s

electron mass $m_e = 9.11 \times 10^{-31}$ kg = 5.4858×10^{-4} u = 0.511 MeV/c^2

neutron mass $m_n = 1.6749 \times 10^{-27}$ kg = 1.008665 u = 939.573 MeV/c^2

proton mass $m_p = 1.6726 \times 10^{-27}$ kg = 1.0072765 u = 938.280 MeV/c^2

1year = 3.156×10^7 s

nuclear radius, $R \approx r_0 A^{1/3}$, where $r_0 = 1.2$ fm

fine structure constant, $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}$

$\hbar c = 197$ MeVfm

The table of nuclear properties is provided in the next page.

Nuclide	Z	A	Atomic mass (u)	I^π	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	0^+	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8 s (β^-)
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.00000	0^+	99.89%
N	7	15	15.00109	$1/2^-$	0.366%
N	7	18	18.014081	1^-	0.63 s
O	8	15	15.003065	$1/2^-$	122 s
O	8	16	15.994915	0^+	99.76%
O	8	18	17.999160	0^+	0.204%
F	9	18	18.000937	1^+	110.0 min
Ne	10	20	19.992436	0^+	90.51%
Ne	10	22	21.991383	0^+	9.33%
Na	11	22	21.994434	3^+	2.60 yrs
Mg	12	21	21.000574	0^+	3.86 s
Al	13	27	26.981539	$5/2^+$	100.0%
Si	14	30	29.973770	0^+	3.10%
Si	14	32	31.974148	0^+	105 yrs
P	15	30	29.978307	1^+	2.50 min
P	15	32	31.971725	1^+	14.3 days
S	16	32	31.972071	0^+	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 days
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 days (β^-)
Sc	21	47	46.952409	$7/2^-$	3.35 days (β^-)
Fe	26	56	55.934439	0^+	91.8%
Fe	26	60	59.934078	0^+	1.5 Myrs
Co	27	60	59.933820	5^+	5.27 yrs
Ni	28	60	59.930788	0^+	26.1%
Ni	28	64	63.927968	0^+	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 hrs (β^-)
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	1^+	12.7 hrs
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	0^+	48.6%
Ru	44	104	103.905424	0^+	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44 hrs (β^-)
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 yrs (β^-)
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 days (β^-)
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%

Question 1

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(a) Calculate the approximate density of nuclear matter in kg/m^3. Hint: Use the given expression for the nuclear radius. (5)

(b) How does this compare with the atomic density? Hint: Assume an atomic radius to be of the order of one Angstrom. (5)

(c) Certain radioactive nuclei emit α particles. Lets assume that α particles are emitted with kinetic energy 4 MeV.

i. What is their velocity in m/s? (4)

ii. Are relativistic effects negligible? (1)

iii. What is the closest that such an α particle can get to the center of an Au nucleus? Hint: Assume a head on collision that is prevented by the repulsive coulomb interaction? (10)

Question 2

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(a) In this problem we compare the binding of ^{4}He with ^{3}He and ^{16}O with ^{15}N.

i. Calculate the binding energy per nucleon for ^{4}He (3)

ii. Calculate the binding energy of the last neutron in ^{4}He. How does it compare with the binding energy per nucleon of this nucleus? **Note**: Mass of ^{3}He is 3.016029u. (4)

iii. Calculate the binding energy per nucleon for ^{16}O (3)

iv. Calculate the binding energy of the last neutron in ^{16}O. How does it compare with the binding energy per nucleon of this nucleus? **Note**: Mass of ^{15}O can be found page 3. (4)

v. What does this tell you about the stability of ^{4}He relative to ^{3}He, and of ^{16}O relative to ^{15}N? (2)

(b) Consider an α decay process in which the parent nucleus is originally at rest.

i. Show that the kinetic energy of the α, E_α particle is related to the Q-value by (6)

$$E_\alpha = \frac{M_D}{M_\alpha + M_D} Q,$$

where $M_i, \quad i \in \{\alpha, D\}$ are the masses of the α particle and the daughter nucleus respectively. Hint: Use momentum and energy conservation, and ignore relativistic effects.

ii. Compare the energy carried away by the α to that carried away by the recoiling nucleus at low values of A to that at high values of A. (3)

Question 3

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The semi empirical mass formula for the mass of a nucleus is given by

$$M(A,Z) = Zm_p + (A-Z)m_n - a_vA + a_sA^{2/3} + a_cZ^2A^{-1/3} + a_a(A-2Z)^2 + \delta,$$

where the symbols have their usual meaning. Use $a_v = 15.8$, $a_s = 18.0$, $a_c = 0.72$, $a_a = 23.5$ and $\delta = \pm a_pA^{-2/3}$ *or* 0 with $a_p = 33.5$. (Neglect the δ term)

(a) Show explicitly that for fixed A, $M(A,Z)$ has minimum value. **Note** that the actual value is not required. (5)

(b) Use the mass formula to determine the most stable nucleus with A=16 and for A=208. **Hint**: Differentiate M(A,Z) with respect to Z while A is fixed. (6)

(c) Use the mass formula to determine the binding energy per nucleon for ${}^{8}_{4}\mathrm{Be}$ and ${}^{12}_{6}\mathrm{C}$. Are they stable? (8)

(d) Experimentally ${}^{8}_{4}\mathrm{Be}$ is known to be unstable, contrary to the result above. Assume ${}^{8}_{4}\mathrm{Be}$ to be made out of two bound α particles: By calculating the Q-value of the processes ${}^{8}_{4}\mathrm{Be} \to \alpha + \alpha$, and ${}^{12}_{6}\mathrm{C} \to \alpha + \alpha + \alpha$ explain why this isotope of Beryllium is unstable. **Note**: Mass of Beryllium-8 is 8.0053051u. (6)

Question 4

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(a) Consider the single particle shell model, in which the first few energy states are given by: $1S$; $1P_{3/2}$, $1P_{1/2}$; $1D_{5/2}$, $2S$, $1D_{3/2}$; $1F_{7/2}$, etc.

i. Which values of orbital angular momentum give odd parity and which give even parity? (1)

ii. Use the shell model to predict the spin and parity of $^{23}_{11}$Na, $^{35}_{17}$Cl and $^{41}_{20}$Ca. (6)

iii. Considering that protons and neutrons have intrinsic dipole moments $2.79\mu_N$ and $-1.91\mu_N$, use the shell model to predict the dipole moment of $^{23}_{11}$Na, $^{35}_{17}$Cl and $^{41}_{20}$Ca. (3)

iv. Do you expect the shell model to give correct spin, parity and dipole moments predictions for $^{23}_{11}$Na, $^{35}_{17}$Cl and $^{41}_{20}$Ca? Explain. (3)

(b) Specify any additional particles needed in the following *weak* reactions to assure the conservation of lepton number.

i. $e^- + ^A_Z X \rightarrow ?$ (3)

ii. $\nu + n \rightarrow ?$ (3)

iii. $^A_Z X \rightarrow ^A_{Z-1} Y + ?$ (3)

iv. $\bar{\nu}_e + p \rightarrow ?$ (3)

Question 5

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(a) Consider the mass fomula from question 3; We can use it to deduce a lower limit for the value of Z for the fission process. Consider a nucleus (A,Z) which undergoes fission into roughly two equal parts $2(A/2,Z/2)$. **Note**: Ignore the a_a and δ terms which are a result of quantum effects.

i. Calculate the binding energy difference $\Delta B = B(A,Z) - 2B(A/2,Z/2)$ and show that (8)

$$\Delta B = a_s A^{2/3}\left(1 - 2^{1/3}\right) + a_c \frac{Z^2}{A^{1/3}}\left(1 - 2^{-2/3}\right).$$

ii. Substitute for a_2 and a_3 to obtain ΔB in terms of A and Z. (4)

iii. For what values of Z is $\Delta B > 0$? (That should give the lower limit for fission to occur.) (4)

(b) The activity of a certain material decreases by a factor of 8 in a time interval of 30 days.

i. What is its half life? (3)

ii. What is its mean life? (3)

iii. What is its disintegration constant? (3)