159

UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2016/2017

TITLE OF PAPER: NUCLEAR PHYSICS

COURSE NUMBER: P442

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT TWO PAGES WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Useful Data:

155

1 unified mass unit $(u) = 1.6605 \times 10^{-27}$ kg $= 931.5$ MeV$/c^2$

Planck's constant $h = 6.63 \times 10^{-34}$ Js

Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K

Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1}

Speed of light (vacuum) $c = 3.0 \times 10^8$ m/s

electron mass $m_e = 9.11 \times 10^{-31}$ kg $= 5.4858 \times 10^{-4}$ u $= 0.511$ MeV$/c^2$

neutron mass $m_n = 1.6749 \times 10^{-27}$ kg $= 1.008665$ u $= 939.573$ MeV$/c^2$

proton mass $m_p = 1.6726 \times 10^{-27}$ kg $= 1.0072765$ u $= 938.280$ MeV$/c^2$

1year $= 3.156 \times 10^7$ s

nuclear radius, $R \approx r_0 A^{1/3}$, where $r_0 = 1.2$ fm

fine structure constant, $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}$

$\hbar c = 197$ MeVfm

The table of nuclear properties is provided in the next page.

Nuclide	Z	A	Atomic mass (u)	I^π	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	0^+	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8 s (β^-)
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.00000	0^+	99.89%
N	7	15	15.00109	$1/2^-$	0.366%
N	7	18	18.014081	1^-	0.63 s
O	8	15	15.003065	$1/2^-$	122 s
O	8	16	15.994915	0^+	99.76%
O	8	18	17.999160	0^+	0.204%
F	9	18	18.000937	1^+	110.0 min
Ne	10	20	19.992436	0^+	90.51%
Ne	10	22	21.991383	0^+	9.33%
Na	11	22	21.994434	3^+	2.60 yrs
Mg	12	21	21.000574	0^+	3.86 s
Al	13	27	26.981539	$5/2^+$	100.0%
Si	14	30	29.973770	0^+	3.10%
Si	14	32	31.974148	0^+	105 yrs
P	15	30	29.978307	1^+	2.50 min
P	15	32	31.971725	1^+	14.3 days
S	16	32	31.972071	0^+	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 days
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 days (β^-)
Sc	21	47	46.952409	$7/2^-$	3.35 days (β^-)
Fe	26	56	55.934439	0^+	91.8%
Fe	26	60	59.934078	0^+	1.5 Myrs
Co	27	60	59.933820	5^+	5.27 yrs
Ni	28	60	59.930788	0^+	26.1%
Ni	28	64	63.927968	0^+	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 hrs (β^-)
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	1^+	12.7 hrs
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	0^+	48.6%
Ru	44	104	103.905424	0^+	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44 hrs (β^-)
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 yrs (β^-)
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 days (β^-)
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%

Question 1

157

(a) Describe the experiment that was used to prove that the plum-pudding model of the atom is not correct. Highlight the results of the experiment and how those results disprove the model. (9)

(b) Before the discovery of the neutron, it was proposed that the penetrating radiation produced when beryllium was bombarded with α particles consisted of high-energy γ−rays produced in reactions such as $\alpha +^{9} \mathrm{Be} \rightarrow^{13} \mathrm{C} + \gamma$

  i. Calculate the Q-value for this reaction. (4)

  ii. If 5MeV α particles are incident on ^{9}Be, calculate the energy of the carbon nucleus and hence, determine the energy of the radiation assuming it is emitted as a single photon. (neglect the momentum of the gamma ray relative to the carbon nucleus) (12)

Note: $m(^{4}\mathrm{He}) = 4.0026\mathrm{u}$, $m(^{9}\mathrm{Be}) = 9.0122\mathrm{u}$, $m(^{13}\mathrm{C}) = 13.0034\mathrm{u}$.

158

Question 2

The deuteron has total angular momentum $J = 1$ so that the ground state is a triplet state. The ground state is not a pure S state, i.e $L = 0$ state, and is formed by an admixture of the triplet S-state ($^3S_1$) and the triplet D(L=2)-state ($^3D_1$),

$$\Psi = \alpha\psi(^3S_1) + \beta\psi(^3D_1).$$

Here $|\alpha|^2 + |\beta|^2 = 1$

(a) What is the value of the spin S for the deuteron? (2)

(b) Explain why the P-state, i.e $L = 1$ state is not compatible with the triplet S-state. (3)

(c) Given that the magnetic moment of deuteron is given by $\mu_D = \mu_p + \mu_n + \frac{1}{2}\mu_N \mathbf{L} = g\mu_N \mathbf{J}$, show that (8)

$$g\mathbf{J} = \frac{1}{2}(g_p + g_n)\mathbf{S} + \frac{1}{2}(g_p - g_n)(\mathbf{s}_p - \mathbf{s}_n) + \frac{1}{2}\mathbf{L},$$

where $\mu_i = g_i\mu_N \mathbf{s}_i$, $\mathbf{S} = \mathbf{s}_p + \mathbf{s}_n$.

(d) Show that $\langle \mathbf{s}_p - \mathbf{s}_n \rangle = 0$ in triplet states. Hence show that (8)

$$g\mathbf{J} = \bar{g}\mathbf{S} + \frac{1}{2}\mathbf{L},$$

where $\bar{g} = \frac{1}{2}(g_p + g_n)$.

(e) Show that $g\mathbf{J}^2 = \bar{g}\mathbf{S}\cdot\mathbf{J} + \frac{1}{2}\mathbf{L}\cdot\mathbf{J}$. (4)

Question 3

'59

(a) A by-product of some fission reactors is ^{239}Pu, which is an α-emitter with a half life of 24120 years. Consider $1kg$ of ^{239}Pu at $t = 0$, [Atomic mass of ^{239}Pu = 239.052163u.

i. What is the number of ^{239}Pu nuclei at $t = 0$? (2)

ii. What is the initial activity? (2)

iii. For how long would you need to store Plutonium until it has decayed to a safe activity level of 0.1 Bq? Give your answer in years. (3)

(b) Radionuclides are useful sources of small amounts of energy in space vehicles, remote communication stations, heart pacemakers, etc. Calculate the initial power available from a gram of ^{210}Po, an α-emitter with an energy of 5.30 MeV and a half life of 138 days. Give your answer in Watts. [Atomic mass of $^{210}_{84}$Po = 209.982848u] (5)

(c) In stars that are slightly more massive than the Sun, hydrogen burning is carried out mainly by the CNO cycle, whose first step is $p +^{12}_{6} \mathrm{C} \rightarrow^{13}_{7} \mathrm{N} + \gamma$. Estimate the energy of the γ, assuming the two initial nuclei are essentially at rest. Justify any simplifying assumptions you make. [Atomic masses: $^{1}_{1}$H = 1.007825u, $^{12}_{6}$C = 12.00000u, $^{13}_{7}$N = 13.005739u] (4)

(d) Consider the nuclear fission reaction $n +^{235}_{92} \mathrm{U} \rightarrow^{141}_{56} Ba +^{92}_{36} \mathrm{Kr} + 3n$.

i. Calculate the energy released (in MeV) in the reaction. [Atomic masses: $^{235}_{92}$U = 235.043915u, $^{92}_{36}$Kr = 91.8973u, $^{141}_{56}$Ba = 140.9139u and neutron mass is 1.008665u] (4)

ii. You wish to run a 1000 MW power reactor using $^{235}_{92}$U fission. How much $^{235}_{92}$U is required for one day's operation? (5)

Question 4

160

(a) Classify the following β decays as allowed, first forbidden, Fermi or Gamow-Teller transitions: (15)

1. $^{3}\text{H} \rightarrow ^{3}\text{He}$
2. $^{14}\text{O}(0^+) \rightarrow ^{14}\text{N}^*(0^+)$
3. $^{47}_{21}\text{Sc}(\frac{7}{2}^-) \rightarrow ^{47}_{22}\text{Ti}^*(\frac{7}{2}^-)$
4. $^{36}_{16}\text{S}(0^+) \rightarrow ^{36}_{17}\text{Cl}(2^-)$
5. $^{152}_{63}\text{Eu}^*(0^-) \rightarrow ^{152}_{62}\text{Sm}(0^+)$

(b) A nucleus in an excited state will most likely de-excite through photon emmission, i.e gamma decay. Describe two other processes that lead to de-excitation of excited nuclei. (4)

(c) Show that in the de-excitation of a nucleus via gamma radiation the energy of a gamma photon, E_γ, is given (6)

$$E_\gamma = E\left(1 - \frac{E}{2Mc^2}\right),$$

where E is the energy of the excited state and M is the mass of the nucleus.

Question 5

161

(a) Given that the mass attenuation coefficients for a bone and tissue are $\mu_m(bone) = 0.32$ and $\mu_m(tissue) = 0.2$ for 60keV γ rays. Consider a piece of bone embedded inside the tissue. Let the intensity of radiation that passes through the tissue be I_1 and the intensity of the radiation passing through tissue and bone be I_2. Calculate the thickness of the bone assuming that $I_1/I_2 = 2$. (8)

(b) Discuss three modes by which a photon can interact with matter. (6)

(c) Discuss the essential features of the strong nuclear force. (4)

(d) Show that the decay $n \rightarrow p + e^-$ cannot conserve angular momentum. (3)

(e) Write short notes on the following.

i. Internal conversion (2)

ii. Bremsstrahlung (2)