UNIVERSITY OF SWAZILAND

/62

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2016 /2017

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P 461

TIME ALLOWED : THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS.

APPENDICES 1 AND 2 CONTAIN DEFINITE INTEGRALS AND PHYSICAL CONSTANTS.

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163

Question one

(a) (i) Explain briefly what is meant by *statistical weight* of a system of particles. (3 marks)

(ii) What is the significance of *statistical weight* as regards the properties of the system? (2 marks)

(b) (i) Four coins marked a, b, c, and d are tossed. If the number of heads (H) and the number of tails (T) obtained in a toss define a macrostate.

(a) Write down all the possible macrostates and (2 marks)
(b) Calculate the number of microstates corresponding to each of the above macrostates. (4 marks)

(ii) Assuming the number of coins is increased from 4 to 8, what would be the maximum number of microstates that can be obtained in a toss. (2 marks)

Hint: $W = \dfrac{N!}{\prod_s n_s!}$

(c) (i) What is meant by *degeneracy* of an energy level? (2 marks)

(ii) Find the degeneracy of an energy level having energy $E = kn^2$, k being a constant and $n^2 = n_x{}^2 + n_y{}^2 + n_z{}^2 = 14$.
n_x, n_y, n_z are quantum numbers corresponding to a quantum state. (3 marks)

(d) (i) Define *density of states* of a system of particles. (2 marks)

(ii) Calculate the *density of states* at an energy level of 2.06 eV for a system of fermions having volume 10^{-4} m^3. (5 marks)

Question Two

164

(a) The Maxwell-Boltzmann distribution function for a system of classical particles in thermal equilibrium is

$$n_s = g_s \exp(\alpha + \beta \varepsilon_s)$$

where the symbols have their usual meanings.

A classical non-degenerate system has 2000 particles arranged in 3 energy levels having energies 1 unit, 2 units and 3 units. The total energy of the system is 2600 units.

(i) Use the above distribution function to find the values of α and β for the system. (9 marks)

(ii) Use these values and the distribution function to find the occupation of the energy levels. (3 marks)

(iii) Verify your results numerically by finding the total number and total energy of the system. (2 marks)

(b) The differential form of Maxwell-Boltzmann distribution function in terms of speed is

$$n(v)dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT} v^2 dv$$

Use this equation to obtain

(i) the mean speed and (6 marks)

(ii) the most probable speed of the molecules in a classical gas. (5 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

166

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan- Boltzmann constant	σ	5.67×10^{-8} Wm^{-2}K^{-4}
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of $_2^4$He atom		6.65×10^{-27} kg
Mass of $_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 L mol^{-1}