UNIVERSITY OF SWAZILAND 15

FACULTY OF SCIENCE AND EGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2016/2017

TITLE O F PAPER: MECHANICS

COURSE NUMBER: PHY211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS.

EACH QUESTION CARRIES 25 MARKS.

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN.

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE.

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE CHIEF INVIGILATOR.

QUESTION 1

16

(a) The acceleration of a body moving in one dimension is given by $a = g - \frac{v}{\tau}$, where g is the acceleration due to gravity, v is the instantaneous velocity, and τ is a constant.

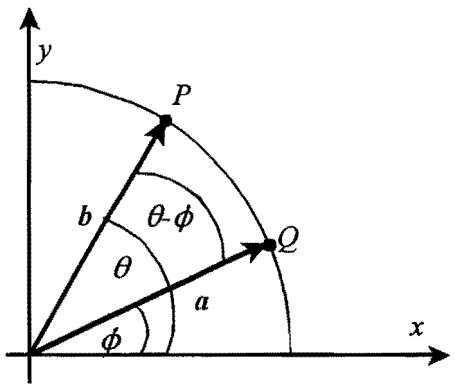
i. Find the velocity of the body as a function of time, and determine its value after a very long time. **(5 marks)**

ii. Find the displacement of the body as a function of time. **(4 marks)**

iii. What is the nature of the displacement function after very long times? **(2 marks)**

(b) Let $\hat{a}$ and $\hat{b}$ be unit vectors in the x-y plane making angles θ and ϕ with the x axis, respectively. Use your knowledge of vectors to prove the trigonometric identity

$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi.$$



(6 marks)

Figure 1.

(c) A speed boat moves towards a bridge with a velocity of 6.00 m/s. A student in the boat sees a friend on the bridge and decides to throw an object to the friend. The student throws the object vertically upward with a velocity v_{0y}, while the boat is a horizontal distance of 8.00 m from the edge of the bridge. The bridge is 7.00 m high. Find the vertical velocity v_{0y} with which the object leaves the hands of the student in the boat. **(8 marks)**

QUESTION 2

17

(a) A bucket of water can be whirled in a vertical circular path such that the water does not spill when it is at its highest point and upside down. Explain why this happens. You could use the aid of a diagram. **(5 marks)**

(b) How does a centrifuge separate particles of different densities in a liquid mixture? **(5 marks)**

(c) A student of mass 64.0 kg makes an elevator with a single pulley. He constructs a frame to support his body and attaches a rope to it that goes over the pulley attached above. The mass of the frame is 12.0 kg. To operate the make shift elevator, the student pulls downward on the other dangling rope. This action makes him accelerate upward at a rate of 2.00 m/s^2. What is the tension in the rope? **(5 marks)**

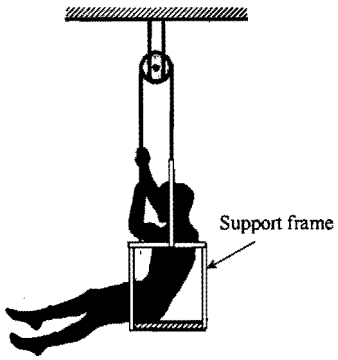


Figure 2.

(d) Mass m is whirled at instantaneous speed v on the end of a string of length R. The motion is in a vertical plane in the gravitational field of the Earth. The forces on m are the weight $W = mg$ down and the string force T toward the center. The string makes instantaneous angle θ with the horizontal.

i. Find the tension T as a function of g, m, R, v and θ. **(6 marks)**

ii. What is the requirement for the velocity of the mass at the highest point in order to complete full circles? **(4 marks)**

QUESTION 3

18

(a) A projectile of mass 255 kg is launched with a velocity of 90.0 m/s at an angle of 40.0° with the horizontal. When it reaches its highest point it breaks into two main pieces through some internal explosion. The masses of other fragments are considered negligible. One piece has a mass of 205 kg while the other has a mass of 50.0 kg. The larger piece is stopped by the explosion and free falls vertically downward. How far from the launch point does the smaller piece land? let the horizontal distance between the launch point and the point where the projectile explodes be L. **(8 marks)**

(b) Find the centre of mass of fraction of a uniform thin disc of radius R subtended by an angle of 30.0° with one of its sides lying along the x-axis as shown in Figure 3. **(5 marks)**

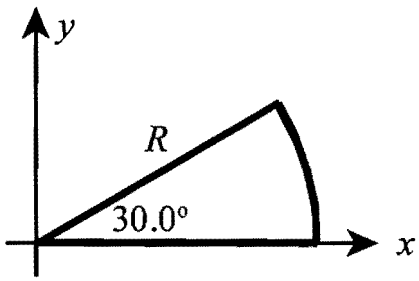


Figure 3.

(c) A freight car of mass M_C contains a mass of sand M_s. At $t = 0$ a constant horizontal force of magnitude F is applied in the direction of rolling and at the same time a port in the bottom is opened to let the sand flow out at the constant rate $\frac{dm}{dt} = b$.

i. Find the speed of the freight car as a function of M_C, M_s, b and t. **(10 marks)**

ii. What is the final velocity of the freight car. **(2 marks)**

QUESTION 4

(a) A small block of mass m starts from rest and slides along a frictionless loop the loop as shown in Figure 6. What should be initial height z, so that the mass pushes against the top of the loop at a with a force equal to its weight? **(7 marks)**

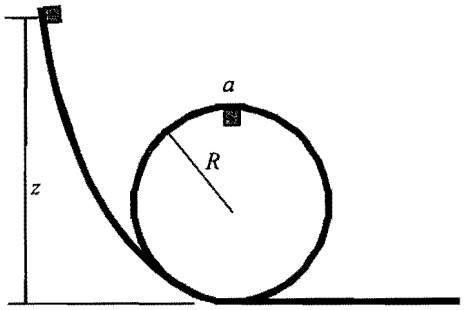


Figure 6.

(b) The force $\vec{F} = A(xy\hat{\imath} + y^2\hat{\jmath})$, where A is a constant with appropriate unit, is used to move a particle from (0,0) to (0,1), along two paths. The first path a goes directly from (0,0) to (0,1) and the second path is along the three segments b, c and d (0,0) to (1,0), (1,0) to (1,1) and (1,1) to (0,1), respectively. See Figure 7. Comment on the results obtained. **(7 marks)**

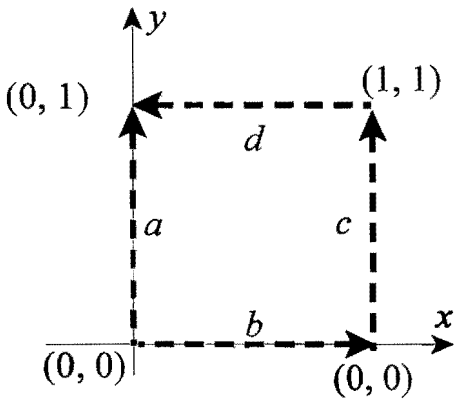


Figure 7.

(c) A potential function for a particle of mass m moving along the x-axis is given by

$$U(x) = -A\left[\left(\frac{x}{\alpha}\right)^3 - \left(\frac{x}{\alpha}\right)^2\right] \quad ,$$

where A and α are constants with appropriate units.

i. Determine the force acting on the particle. **(2 marks)**

ii. Find the equilibrium points. **(3 marks)**

iii. Determine the stability of the equilibrium points. **(4 marks)**

iv. Find the angular frequency of oscillation about the stable equilibrium point. **(2 marks)**

QUESTION 5

20

(a) Find the moment of inertia of a uniform thin rod of mass M and length L about an axis perpendicular to its length through on of its end points. **(4 marks)**

(b) A uniform sphere of mass M and radius R and a uniform cylinder of mass M and radius R are released simultaneously from rest at the top of an inclined plane. Both roll without slipping and the moment of inertia about an axis of symmetry for the drum and sphere are $\frac{1}{2}MR^2$ and $\frac{2}{5}MR^2$, respectively. Which body reaches the bottom first if they both roll without slipping. **(9 marks)**

(c) A uniform cylindrical drum of radius b and mass M rolls without slipping down a plane inclined at an angle θ as shown in Figure 10. Find its acceleration down the plane in terms of the acceleration due to gravity g and the angle θ, using torque and angular momentum.
(12 marks)

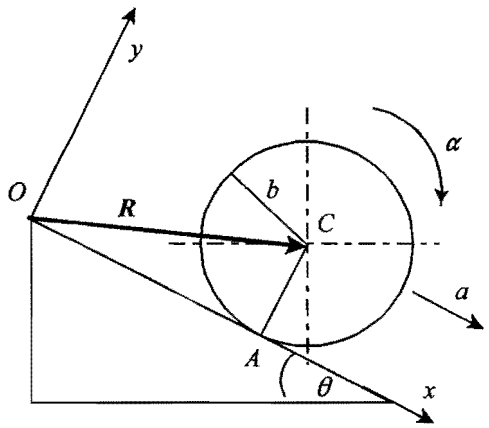


Figure 10.