UNIVERSITY OF SWAZILAND

21

FACULTY OF SCIENCE AND EGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY/RESIT EXAMINATION 2016/2017

TITLE OF PAPER: MECHANICS

COURSE NUMBER: P211/PHY211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: — ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

22

(a) Derive the basic kinematic equation, $v^2 = {v_0}^2 + 2a(x - x_0)$. **(5 marks)**

(b) A body is projected with a velocity v_0 at an angle θ with the horizontal on a flat field. Find

i. the maximum height h reached, and **(6 marks)**

ii. the maximum range R, **(4 marks)**

of the projectile both in terms of v_0, θ and g.

(c) Derive expressions for $\frac{d\hat{r}}{dt}$ and $\frac{d\hat{\theta}}{dt}$ for plane polar coordinates, and use the results to write an expression for the velocity in in these coordinates. **(10 marks)**

QUESTION 2

2.3

(a) Three blocks ($m_1 = 8.00$ kg, $m_2 = 6.00\ kg$, and $m_3 = 4.00$ kg) are arranged on a frictionless table as in Figure 1. A force $F = 24.0$ N is applied to the mass m_1.

(i) What is the acceleration of the system? **(2 marks)**

(ii) Find the force between m_1 and m_2 and between m_2 and m_3. **(6 marks)**

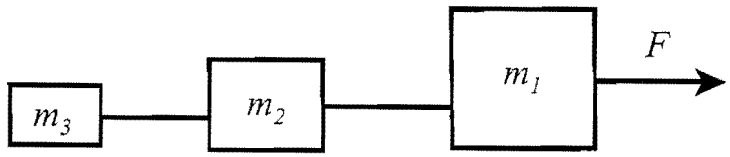

Figure 1.

(b) In Figure 2, A and B are points on a fixed vertical shaft at a distance $6l$ apart. A particle of mass m is attached to A and B by light inextensible strings of length $5l$. The particle moves in a horizontal circle with angular velocity ω.

i. Make a correct resolved force diagram for the particle from which you can make useful equations. **(4 marks)**

ii. Determine the tensions T_1 (in upper string) and T_2 (in lower string) in terms of m, g, l, θ and ω. **(8 marks)**

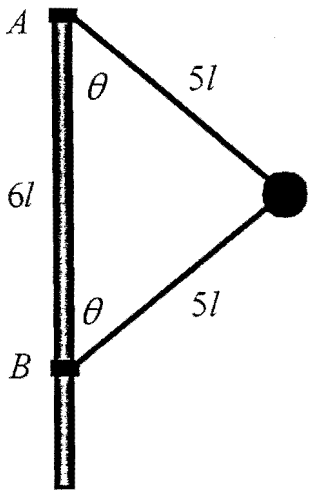

Figure 2.

(c) The International space station travels at speed v. The mass of the earth is M_E, the radius of the Earth is R_E, and the universal gravitational constant is G. Find the radius of orbit of the space station from the centre of the earth in terms of g, R_E and v, where g is the acceleration due to gravity at the earth's surface. **(5 marks)**

QUESTION 3

24

(a) Find the centre of mass of a fraction of a uniform thin disc of inner radius R_1 and outer radius R_2 subtended by an angle of 180.0° as shown in Figure 3. **(8 marks)**

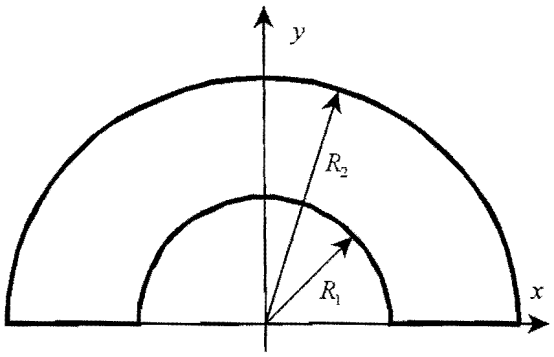

Figure 3.

(b) A projectile of mass $4m$ is launched at an angle with the horizontal. When it reaches its highest point it undergoes an internal explosion where a piece of mass m moves forward and lands a distance $3L$ from the launching point, while the piece of mass $3m$ lands a distance L' from the launching point. Find the distance L' where the larger piece lands. The explosion occurs a horizontal distance L from the launching point. **(7 marks)**

(c) A space craft encounters a stationary dust cloud where some particles of the dust stick to it at a rate $dm/dt = bM$, where M is the instantaneous mass of the rocket and accumulated dust, and b a constant with appropriate unit. The rocket is also acted upon by an external force proportional to its instantaneous mass in the direction of its motion. This force is given by $F = \gamma M$, where γ is a constant with appropriate unit.

i. Find the speed of the rocket as a function of b, γ and t. **(8 marks)**

ii. What is the final velocity of the rocket? **(2 marks)**

QUESTION 4

25

(a) A particle of mass m is projected upward with a velocity v_0 in an inverse square field given by $-\frac{A}{r^2}$, from a radius r_0, where A is a constant. Use the work energy theorem to find the initial velocity for the particle to escape the field in terms of A, m and r_0.
(6 marks)

(b) A particle is acted upon by the force $F = x\hat{\imath} + y\hat{\jmath}$ and is used to move a particle along the path that form a semicircle from $(x = 0, y = 0)$ to $(x = 0, y = 2R)$ as shown in Figure 4. Find the work done by the force. **(8 marks)**

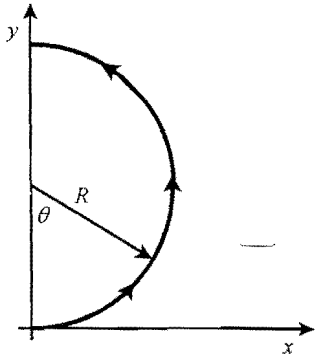

Figure 4.

(c) A particle of effective mass m is acted upon by a force F under a potential $U(r) = -U_0ar^2 + U_0br^4$, where a and b are positive constants with appropriate units.

i. Find the force acting on the particle. **(2 marks)**
ii. Find the equilibrium point(s) for the particle. **(3 marks)**
iii. Determine the stability of the equilibrium point(s). **(3 marks)**
iv. Determine the frequency of small oscillations about the equilibrium point(s). **(3 marks)**

QUESTION 5

26

(a) Find the moment of inertia of a nonuniform stick of length L about an axis through the lighter end. The length density of the stick is given by $\lambda = \frac{\lambda_0}{L}x$ where λ_0 is a constant. **(7 marks)**

(b) If you remove two eggs from the refrigerator, one is hardboiled and the other is raw, and you set them spinning on the floor with the same initial torque, explain which one would rotate uniformly and faster and which one will rotate slowly and wobbly? **(4 marks)**

(c) Show that the torque $\vec{\tau}$ and the angular momentum $\vec{L}$ are related by the equation: $\vec{\tau} = \frac{d\vec{L}}{dt}$. **(5 marks)**

(d) A uniform sphere of mass M and radius R and a uniform cylinder of the same mass M and radius R are released simultaneously from rest at the top of an inclined plane with friction. The inclined plane makes an angle θ with the horizontal. Find the acceleration of each body to determine which body arrives at the bottom first. The moments of inertia for the cylinder and sphere are $\frac{1}{2}MR^2$ and $I_s = \frac{2}{5}MR^2$, respectively. **(9 marks)**