UNIVERSITY OF SWAZILAND

58

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2016/2017

TITLE OF THE PAPER: COMPUTATIONAL PHYSICS I

COURSE NUMBER: PHY 282/P262

TIME ALLOWED:
SECTION A: ONE HOUR
SECTION B: TWO HOURS

INSTRUCTIONS:
THERE ARE TWO SECTIONS IN THIS PAPER:

- **SECTION A** IS A WRITTEN PART. ANSWER THIS SECTION ON THE ANSWER BOOK. IT CARRIES A TOTAL OF **30** MARKS.
- **SECTION B** IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. IT CARRIES A TOTAL OF **70** MARKS.

Answer **all** the questions from Section A and **all** the questions— from Section B. Marks for different sections of each Question are shown in the right hand margin.

THE PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

Section A – Use a pen and paper to answer these questions

59

Question 1

(a) In simple words, what is an interpreted programming language.

[3 marks]

(b) What are the tasks that MAPLE preferably not be used for

(i) `Developing large software projects`
(ii) `Developing numerical algorithms`
(iii) `Symbolic calculations`
(iv) `Developing an application that has to run as fast as possible`

[2 marks]

(c) In numerical analysis, the Euler method ....

(i) `is the first-order numerical procedure for solving initial value problem for ordinary differential equations.`
(ii) `is a third-order numerical procedure for solving initial value problem for ordinary differential equations`
(iii) `is very fast and stable and thus should be always be the first method to try when solving ordinary differential equations.`
(iv) `All the above`

[2 marks]

(d) What values of x and y are given out after the following statements have been executed?

```
x:=1;y:=1;
s:=x+y;
x:=x+x/s;
s:=x+y;
y:=y+x/s;
x=x+y*s;
```

[3 marks]

Question 2

60

(a) A discharging capacitor can be described by the iterative equation

$$Q_{i+1} = Q_i - \Delta t \frac{Q_i}{\tau}$$

where $Q_i \equiv Q(t_i)$ is charge at time $t_i = i \cdot \Delta t$ and τ is the time constant. Assume that the time step $\Delta t = 0.1$s, the initial charge $Q_0 = 1000\mu$F, and $\tau = 1$s. Write a program that calculates the charge in capacitor Q_i at time t_i, for i =0...100.

[4 marks]

(b) In nuclear physics the semi-empirical mass formula gives an approximate value for the binding energy B of a nucleus with atomic number Z and a mass number A:

$$B = a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4 (A - 2Z)^2 / A + a_5 A^{-1/2},$$

where, in units of MeV, the constants are $a_1 = 15.67$, $a_2 = 17.23$, $a_3 = 0.75$, $a_4 = 93.2$, and

$$a_5 = \begin{cases} 12.0 & \text{if Z and A -Z are both even,} \\ -12.0 & \text{if Z and A-Z are both odd,} \\ 0 & \text{otherwise} \end{cases}$$

Write a program that takes as its input the values of A and Z and returns the binding energy per energy for the corresponding atom.

[6 marks]

Question 3

61

(a) The program below is supposed to convert the temperature of boiling water (212 °F) from Fahrenheit to Celsius (°C) using the conversion formula

$$C = \frac{5}{9}(f - 32)$$

but it does not produce the correct result. Discuss briefly what is the output of the program. Fix it such that the originally intended purpose is restored.

```
restart;
Fer_2_Cel:=proc(x)
C:=5/9*f-32;
return C end proc:
Fer_2_Cel(212);
```

[4 marks]

(b) Describe exactly but briefly what is the output of the program:

```
program1:=proc(N)
x:=0;
for i from 0 to N do
x:=x+i**2:
end do:
return x;
program1(3);
```

[3 marks]

(c) The function below is supposed to return 1 if n equals to 0 but it always returns 0. Why? Fix it such that the originally intended purpose is restored.

```
f:=proc(n)
if (n=0) then 1;
end if;
0;
end proc;
```

[3 marks]

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Section B – Practical Part

62

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Question 4

(a) Consider the two functions: $W(\phi) = \sin^4(\phi)$ and $U(\phi) = \sin^3(\phi)\cos(\phi)$.

(i) Plot $W(\phi)$ and $U(\phi)$ on the same graph for $\phi = 0..2\pi$.

[4 marks]

(ii) Calculate the

$$\langle W(\phi)\rangle \text{ and } \langle V(\phi)\rangle,$$

where $\langle ...\rangle = \frac{1}{2\pi}\int_0^{2\pi} ...d\phi$.

[4marks]

(b) Consider an oscillatory potential

$$V(r) = a\cos(r) + b\cos(2r) - cr$$

with $a = 5, b = 1$, and $c = 1.5$.

(i) Plot $V(r)$ for $r = -10..10$.

[3 marks]

(ii) Find the local maximum of $V(r)$ near zero, and the local minimum of $V(r)$ to the left (negative side) of zero.

[6 marks]

(iii) What is the potential energy barrier for moving from one well to the next in this potential?

[3 marks]

Question 5

63

(a) The velocity $v(t) = -9.8t + v_0$ (m/s) and height $h(t) = -4.9t^2 + v_0 t + h_0$ (m) are familiar formulars for an object moving under the influence of gravity, where v_0 and h_0 are the initial velocity and height when $t = 0$ respectively.

Write a Maple **procedure** that calculates the height above the ground and the velocity of an object thrown vertically after a certain number of seconds. The input of the procedure should be the initial height and velocity, as well as the time elapsed. Execute the program with an initial height 100m and an initial velocity of 5 m/s (upwards) over 3 seconds.

[10 marks]

(b) Consider the Matrix

$$\mathbf{M} = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1}$$

(i) Find the eigenvalues λ and eigenvectors α_i of the matrix $\mathbf{M}$.

(ii) Take one set of eigenstates (eigenvalue and the corresponding eigenvector) and show that indeed $M_{ij}\alpha_i = \lambda\alpha_i$.

[10 marks]

Question 6

64

A pendulum can be driven by, for example, exerting a small oscillating force horizontally on the mass. Then the equation of motion for the pendulum becomes

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(\theta) + C\cos(\theta)\sin\Omega t$$

where C and Ω are constants.

(a) Write a program to solve the above equation using the fourth-order RungeKutta method. Solve for θ as a function of time with $l = 1$ m, $g = 9.8\text{m/s}^2$, $C = 2$ s^{-2} and Ω =5 s^{-1} and make a plot of $\theta(t)$ as a function of time from t = 0 to t = 100 s. Start the pendulum at rest, $\theta(0) = 0$ and $\theta'(0) = 0$. *NB: You may need to decompose the above equation into a system of two first order ODEs.*

[15 marks]

(b) Now change the value of Ω, while keeping C the same, to find a value for which the pendulum resonates with the driving force and swings widely from side to side. Make a plot for this case also.

[15 marks]