UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2017/2018

TITLE OF PAPER: SOLID STATE PHYSICS

COURSE NUMBER: P412

TIME ALLOWED: THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One

(a) (i) Define *unit cell* of a crystal. (2 marks)

(ii) Distinguish between a primitive unit cell and a conventional unit cell. (2 marks)

(iii) Draw the Wigner - Seitz cell of a two-dimensional direct lattice. State whether it is a primitive or a conventional cell. (2 + 1 marks)

(iv) Sodium has a density of 0.968 g/cm^3 and a unit cell side of length 4.29 Å. Determine the lattice type. [Atomic mass of sodium = 22.99 g/mol] (4 marks)

[hint : Calculate the number of atoms/unit cell]

(b) (i) In the diagram of a cubic unit cell, show a (121) and a (212) plane. (2 + 2 marks)

(ii) Calculate the distance between two such planes if the lattice constant is 1 Å. (2 marks)

(iii) A cubic crystal plane has intercepts $3a$, $2a$ and $1a$ along x , y, and z axes where a is the lattice constant. Find the Miller indices of this plane. (2 marks)

(c) (i) What is meant by *packing fraction* of a crystal? (2 marks)

(ii) Determine the packing fraction of a diamond lattice. (4 marks)

Question Two

(a) (i) What is van der Waals-London attractive interaction in inert gas crystals? (3 marks)

(ii) Explain how Pauli's exclusion principle is responsible for the repulsive interaction in inert gas crystals. (3 marks)

(b) Derive an expression for the total lattice energy of a one -dimensional crystal consisting of a line of $2N$ ions of alternating charge $\pm q$ at their equilibrium separation R_o. The repulsive interaction may be assumed to be of the form $\lambda \exp(-r/\rho)$ where λ and ρ are empirical parameters and r is the separation between two ions.

(12 marks)

(c) In an ionic crystal, a line of $2N$ ions of alternating charges $\pm q$ have a repulsive potential energy of the form A/R^n, where R is the distance between adjacent ions. Show that at equilibrium separation R_0 the potential energy,

$$U_{tot} = \frac{-2Nq^2 \ln 2}{R_0}\left(1 - \frac{1}{n}\right).$$

[Given: Madelung constant = 2 ln2] (7 marks)

Question Three

(a) Given below are translation vectors in a direct lattice and a reciprocal lattice respectively:

$\mathbf{T} = n_1\mathbf{a} + n_2\mathbf{b} + n_3\mathbf{c}$, $\mathbf{G} = h\mathbf{A} + k\mathbf{B} + l\mathbf{C}$ where the symbols have their meanings

(i) Write down vectors **A, B** and **C** in terms of **a, b** and **c.** (2 marks)

(ii) Show that exp (i**G.T**) = 1. (4 marks)

(b) (i) A wave of wave vector **k** is incident on a crystal specimen. The diffracted wave has wave vector **k'**. Show that the diffraction condition for constructive interference between the two waves can be written as: **G = Δk**, where **Δk = k' - k** and **G** is a reciprocal lattice vector.

What is the physical meaning of the above condition?

(10 marks)

Given: $n(\bar{r}) = \sum_{G} n_G \exp i\bar{G}.\bar{r}$

(ii) The *geometric structure factor* of a crystal is:

$$S_G = \sum_{j=1}^{s} f_i \exp[-i2\pi(n_1h + n_2k + n_3l)]$$

, where s is the number of atoms in the basis and n_1, n_2, n_3 are fractional coordinates. f is the atomic form factor.

Explain the significance of structure factor as regards the identification of lattice type of crystals using x-ray diffraction. (4 marks)

(iii) Show that in a crystal having bcc lattice, Bragg reflection occurs only from planes with Miller indices adding up to an even number. (5 marks)

Question Four

(a) (i) Explain how an intrinsic sample of silicon can be made n-type or p-type by appropriate doping. (4 marks)

(ii) Explain what is meant by *effective density of states* of a semiconductor. (2 marks)

(iii) The effective density of states in the conduction and valence bands of a semiconductor is given as:

$$N_{c,v} = 2\left(\frac{2\pi mkT}{h^2}\right)^{3/2}$$

where the symbols have their usual meanings. Using the above expression, write down the electron and hole concentrations in the conduction and valence bands. (2 marks)

(iv) Show that the electrical conductivity of an intrinsic semiconductor can be expressed as

$$\sigma_i = A\exp\left(\frac{-E_g}{2kT}\right)$$

where E_g is its band gap and the other symbols have their usual meanings. (4 marks)

(v) Explain how you would use the above expression to find the band gap of a material experimentally. (4 marks)

(b) Calculate: (i) the effective density of states and (6 marks)

(ii) the intrinsic carrier concentration of silicon. (3 marks)

[Effective masses of electrons and holes are 1.1 m_0 and 0.56 m_0, respectively. Band gap of silicon = 1.1 eV]

<u>Question Five</u>

(a) Discuss briefly the *free electron approximation* in metals. (4 marks)

(b) Assume a plane wave $\psi_k(\bar{r}) = \exp i(\bar{k}.\bar{r})$ represents a free electron, where the symbols have their usual meanings. Use the Schrodinger wave equation to obtain its energy eigenvalues, ϵ_k. (4 marks)

(c) (i) What is meant by *Fermi energy*? (2 marks)

(ii) Use the results in (b) above to show how the Fermi energy is related to the electron concentration and hence (7 marks)

(iii) Derive an expression for the *density of states* of the electrons in a metal. (3 marks)

(d) Calculate the Fermi energy of potassium, given that it has a density of 8.6×10^2 kg m^{-3} and an atomic weight of 39. (5 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} Js
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan- Boltzmann constant	σ	5.67×10^{-8} Wm^{-2} K^{-4}
Atmospheric pressure		1.01 10^{5} Nm^{-2}
Mass of ${}_2^4$ He atom		6.65×10^{-27} kg
Mass of ${}_2^3$ He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 L mol^{-1}