UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2017/2018

TITLE OF PAPER: SOLID STATE PHYSICS

COURSE NUMBER: P 412

TIME ALLOWED : THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One

(a) (i) Distinguish between a primitive unit cell and a conventional unit cell. (2 marks)

(ii) Draw conventional unit cells of face-centred and body-centred cubic lattices of lattice constant *a*. For each lattice, write down the number of lattice points per cell and the volume of the primitive cell. (6 marks)

(iii) What is meant by *packing fraction* of a crystal? Determine the packing fraction of a b.c.c. crystal (2 + 3 marks)

(b) (i) In the diagram of a cubic unit cell, show a (110) and a (200) plane. (4 marks)

(ii) Calculate the separation between two (110) planes of a cubic crystal in terms of its lattice constant. (2 marks)

(iii) The lattice constant of a b.c.c. crystal is 500 pm. Calculate the radius of one atom. (3 marks)

(iv) The length of a unit cell of gold is 4.08×10^{-8} cm. There are 5.9×10^{22} atoms/cm^3.
(1) Find how many unit cells are in a volume of 1 cm^3.
(2) Find how many atoms are in a unit cell.
(3) Determine the lattice type. (3 marks)

Question Two

(a) (i) State what is meant by *density of states,* in terms of frequency ω, as applied in lattice dynamics. (2 marks)

(ii) By considering the boundary conditions on lattice waves in a cubic crystal, prove that for a three dimensional system, the density of states is given by the expression

$$D(\omega) = \frac{V\omega^2}{2\pi^2 v^3}$$

where V is the volume, and v is the velocity of sound. (8 marks)

(b) (i) What does *Debye approximation* mean? (3 marks)

(ii) Show that by applying Debye approximation, the density of states for a system with one atom per unit cell is given by

$$D(\omega) = \frac{9N\omega^2}{\omega_D^3}$$

where N is the number of atoms and ω_D is the Debye frequency. (7 marks)

(c) Use the result in (b) above to show that the zero point energy of a lattice is given by

$$E = \frac{9N}{8}\hbar\omega_D$$ (5 marks)

Given: the mean energy of a harmonic oscillator is

$$\bar{\varepsilon} = \hbar\omega\left(\frac{1}{2} + \frac{1}{e^{\hbar\omega/kT} - 1}\right)$$

Question Three

(a) Derive the phonon dispersion relation $\omega = \left(\frac{4C}{M}\right)^{1/2} \sin\frac{1}{2}ka$ for a one-dimensional monatomic linear lattice of lattice constant a, atomic mass M and force constant C.

(13 marks)

(b) (i) Draw a sketch showing how the phonon frequency varies with wave vector in the first Brillouin zone. (3 marks)

(ii) What are the values of the frequency for $k = 0$ and $k = \pi/a$?

(2 marks)

(iii) Show that when the phonon wavelength is large compared to the interatomic spacing, the phase velocity, $\frac{\omega}{k} = a\sqrt{\frac{C}{M}}$, where the symbols have their usual meanings. (4 marks)

(c) Calculate the velocity of the elastic waves in a linear lattice of lattice constant 1 Å, force constant 30 N m^{-1}, and atomic mass 10^{-27} kg.

(3 marks)

Question Four

(a) (i) Define *Fermi energy*. (2 marks)

(ii) Derive an expression for the density of states of a system of electrons, given that the Fermi energy:

$$\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$ (6 marks)

where the symbols have their usual meanings.

(iii) Calculate the density of energy states at 2.05 eV energy, for a system of electrons in a volume of 1 cm^3. (8 marks)

(b) (i) Show that the electronic contribution to heat capacity of a metal is proportional to absolute temperature. (6 marks)

(ii) Discuss the heat capacity of metals, explaining the difference, if any, between the above theory and the experimental values. (3 marks)

Question Five

(a) Using silicon as an example, explain how the electrical conductivity of a semiconductor can be increased by doping.

(6 marks)

(b) With the help of an appropriate diagram, derive an expression for the effective density of states in the conduction band of a semiconductor. Assume: $(\epsilon-\epsilon_F) \gg kT$.

(10 marks)

[Given: Fermi -Dirac distribution function: $f(\varepsilon) = \dfrac{1}{e^{(\varepsilon-\varepsilon_F)/kT} + 1}$]

(c) A doped semiconductor has electron and hole concentrations of 2×10^{13} cm^{-3} and 1.41×10^{13} cm^{-3} respectively. Calculate the electrical conductivity of the sample.

(5 marks)

[Take: $\mu_n = 4200$ $cm^2 V^{-1} s^{-1}$ and $\mu_p = 2000$ $cm^2 V^{-1} s^{-1}$]

(d) Discuss, briefly, the process of photoconductivity in semiconductors. (4 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\,\pi^{1/2}}{2}$$

<u>Appendix 2</u>

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	$3.00 \times 10^{8}\ \mathrm{ms^{-1}}$
Plank's constant	h	$6.63 \times 10^{-34}\ \mathrm{Js}$
Boltzmann constant	k	$1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$
Electronic charge	e	$1.61 \times 10^{-19}\ \mathrm{C}$
Mass of electron	m_e	$9.11 \times 10^{-31}\ \mathrm{kg}$
Mass of proton	m_p	$1.67 \times 10^{-27}\ \mathrm{kg}$
Gas constant	R	$8.31\ \mathrm{J\ mol^{-1}\ K^{-1}}$
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	$9.27 \times 10^{-24}\ \mathrm{JT^{-1}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$
Stefan- Boltzmann constant	σ	$5.67 \times 10^{-8}\ \mathrm{Wm^{-2}\ K^{-4}}$
Atmospheric pressure		$1.01\ \ 10^{5}\ \mathrm{Nm^{-2}}$
Mass of ${}_2^4$He atom		$6.65 \times 10^{-27}\ \mathrm{kg}$
Mass of ${}_2^3$He atom		$5.11 \times 10^{-27}\ \mathrm{kg}$
Volume of an ideal gas at STP		$22.4\ \mathrm{L\ mol^{-1}}$