UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2017/2018

TITLE OF PAPER: NUCLEAR PHYSICS

COURSE NUMBER: P442

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT TWO PAGES WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

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Useful Data:

1 unified mass unit (u) = 1.6605×10^{-27} kg = 931.5 MeV/c^2

Planck's constant $h = 6.63 \times 10^{-34}$ Js

Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K

Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1}

Speed of light (vacuum) $c = 3.0 \times 10^8$ m/s

electron mass $m_e = 9.11 \times 10^{-31}$ kg $= 5.4858 \times 10^{-4}$ u = 0.511 MeV/c^2

neutron mass $m_n = 1.6749 \times 10^{-27}$ kg = 1.008665 u = 939.573 MeV/c^2

proton mass $m_p = 1.6726 \times 10^{-27}$ kg = 1.0072765 u = 938.280 MeV/c^2

1year = 3.156×10^7 s

nuclear radius, $R \approx r_0 A^{1/3}$, where $r_0 = 1.2$ fm

fine structure constant, $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}$

$\hbar c = 197$ MeVfm

The table of nuclear properties is provided in the next page.

Nuclide	Z	A	Atomic mass (u)	I^{π}	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	0^+	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8 s (β^-)
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.00000	0^+	99.89%
N	7	15	15.00109	$1/2^-$	0.366%
N	7	18	18.014081	1^-	0.63 s
O	8	15	15.003065	$1/2^-$	122 s
O	8	16	15.994915	0^+	99.76%
O	8	18	17.999160	0^+	0.204%
F	9	18	18.000937	1^+	110.0 min
Ne	10	20	19.992436	0^+	90.51%
Ne	10	22	21.991383	0^+	9.33%
Na	11	22	21.994434	3^+	2.60 yrs
Mg	12	21	21.000574	0^+	3.86 s
Al	13	27	26.981539	$5/2^+$	100.0%
Si	14	30	29.973770	0^+	3.10%
Si	14	32	31.974148	0^+	105 yrs
P	15	30	29.978307	1^+	2.50 min
P	15	32	31.971725	1^+	14.3 days
S	16	32	31.972071	0^+	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 days
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 days (β^-)
Sc	21	47	46.952409	$7/2^-$	3.35 days (β^-)
Fe	26	56	55.934439	0^+	91.8%
Fe	26	60	59.934078	0^+	1.5 Myrs
Co	27	60	59.933820	5^+	5.27 yrs
Ni	28	60	59.930788	0^+	26.1%
Ni	28	64	63.927968	0^+	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 hrs (β^-)
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	1^+	12.7 hrs
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	0^+	48.6%
Ru	44	104	103.905424	0^+	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44 hrs (β^-)
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 yrs (β^-)
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 days (β^-)
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%

Question 1

(a) The de Broglie wavelength is useful in determining whether one will obtain relevant information from scattering processes.

i. The de Broglie wavelength of a particle of mass m is determined from the momentum p using $p = \hbar k = \hbar 2\pi/\lambda$. Show that the de Broglie wavelength can be expressed as follows: (5)

$$\lambda = 2\pi \frac{\hbar c}{mc^2}\sqrt{\frac{mc^2}{2E}}$$

where E is the kinetic energy of the particle.

ii. A nuclear reactor produces fast neutrons (with $E \sim 1MeV$) which are then slowed down to thermal neutrons (with $E \sim 0.025eV$, comparable to their thermal energy at room temperature). In research reactors, both type of neutrons could be selected to exit through a port and used in scattering experiments to study crystals. Crystal lattice spacing is usually a few angstrom, and to get information about the crystal in a scattering experiment, the radiation wavelength should be on the same order of the lattice spacing. Would you select fast or thermal neutrons for scattering experiments on crystals? (6)

iii. At what kinetic energies would electrons be suitable to probe nuclear structure? (2)

(b) List the main physical assumptions that Rutherford made in order to derive the classical differential cross-section formula describing the scattering of α-particles from a thin metal foil target. (12)

Question 2

(a) When we go through the different nuclides, we find that there are certain values of Z and N that are referred to as *magic number* nuclei.

i. What is meant by Magic Numbers? (2)

ii. Give three pieces of experimental evidence for the existence of Magic Numbers. (6)

(b) Consider the shell model, where the first three shells are: (1S), (1P) and (1D,2S)

i. Explain why S states do not split, while all other states get split into two. (2)

ii. Use the shell model to determine spin and parity (J^P) for the ground of $^{15}_{8}O$, $^{7}_{4}Be$ and $^{17}_{8}O$ (9)

(c) Explain why the simple shell model can not be used to predict excited states of even A nuclei. (3)

(d) Explain why the simple shell model can only predict low lying excited states of odd A nuclei but fails for higher excited states. (3)

Question 3

One can follow a few steps to show that for a square well potential in 3-D there is a minimum depth V_0 which will allow for a two body bound state for a system such as Deuteron. In this problem you have to follow similar steps to show that in 1-D there is no minimum depth required for the existence of a bound state.

(a) Explain why the wave function will have either the cosine or sine function for $x < R$. For the rest of the problem we will let $\psi(x) = B\cos(k_1 x)$ for $x < R$. (2)

(b) Explain why the term with a positive exponent is not admissible in the wave function, i.e we take $\psi(x) = Ce^{-k_2 x}$ for $x > R$ when $x > 0$. (2)

(c) Using the boundary conditions that the wave function and its derivative are continuous at the boundary $x = R$, show that. (5)

$$k_2 = k_1 \tan(k_1 R).$$

(d) Using the complete normalized wave function, (equations above) calculate the expectation value of the potential energy. Note that for $x < 0$ you transform x to $-x$ in the wave function described above. (5)

(e) Calculate the expectation value of the kinetic energy (5)

(f) Show that, for a bound state to exist, it must be true that $\langle T\rangle < -\langle V\rangle$. (3)

(g) Finally, show that a bound state will always exist for a square well potential in 1-D. (3)

Note: for $x < R$ we have $\psi(x) = A\sin(k_1 x) + B\cos(k_1 x)$ and for $x > R$ we have $\psi(x) = Ce^{-k_2 x} + De^{k_2 x}$. Here $k_1 = \sqrt{2m(E + V_0)/\hbar^2}$ and $k_2 = \sqrt{-2mE/\hbar^2}$.

Question 4

(a) A by-product of some fission reactors is ^{239}Pu (plutonium-239), which is an α-emitter with a half life of 24120 years. Consider 1 kg of ^{239}Pu at $t = 0$, [Atomic mass of ^{239}Pu = 239.052163 u.

i. What is the number of ^{239}Pu nuclei at $t = 0$? (2)

ii. What is the initial activity? (2)

iii. For how long would you need to store plutonium-239 until it has decayed to a safe activity level of 0.1 Bq? (3)

(b) Radionuclides are useful sources of small amounts of energy in space vehicles, remote communication stations, heart pacemakers, etc. Calculate the initial power available from a gram of ^{210}Po, an α-emitter with an energy of 5.30 MeV and a half life of 138 days. Give your answer in Watts. [Atomic mass of $^{210}_{84}$Po = 209.982848 u] (5)

(c) In stars that are slightly more massive than the Sun, hydrogen burning is carried out mainly by the CNO cycle, whose first step is $\mathrm{p} + ^{12}_{6}\mathrm{C} \rightarrow ^{13}_{7}\mathrm{N} + \gamma$. Estimate the energy of the γ, assuming the two nuclei are essentially at rest. Justify any simplifying assumptions you make. [Atomic masses: $^{1}_{1}H$ = 1.007825 u, $^{12}_{6}$C = 12.00000 u, $^{13}_{7}$N = 13.005739 u] (4)

(d) Consider the nuclear fission reaction $\mathrm{n} + ^{235}_{92}\mathrm{U} \rightarrow ^{141}_{56}\mathrm{Ba} + ^{92}_{36}\mathrm{Kr} + 3\mathrm{n}$.

i. Calculate the energy released (in MeV) in the reaction. [Atomic masses: $^{235}_{92}$U = 235.043915 u, $^{92}_{36}$Kr = 91.8973 u, $^{141}_{56}$Ba = 140.9139 u and neutron mass is 1.008665 u] (4)

ii. You wish to run a 1000 MW power reactor using $^{235}_{92}$U fission. How much $^{235}_{92}$U is required for one day's operation? (5)

Question 5

(a) The differential cross section for Rutherford scattering is proportional to $\sin^{-4}(\theta/2)$ where θ is the scattering angle. Show that this term leads to an infinite cross section in the limit $\theta \to 0$. Explain why, in reality, experimental differential cross sections remain finite as $\theta \to 0$. (5)

(b) The nuclear electric form factor is

$$F(\vec{q}) = \int \rho_{ch}(\vec{r}) \exp(-i\vec{q} \cdot \vec{r}) d^3\vec{r},$$

where ρ_{ch} is the charge density.

i. In the case of spherical symmetry, we have only the radial dependence. Show that $F(\vec{q})$ becomes (5)

$$F(q^2) = \frac{4\pi}{q} \int \rho_{ch}(r) \sin(qr) r dr$$

ii. Assuming that the nuclear charge density is uniform and that the nucleus is a sphere of radius R, obtain an expression for the form factor of a nucleus. (5)

(c) Show that, for high-energy elastic scattering where the projectile rest mass may be ignored, the magnitude of the momentum transfered q from the incident particle is given by (10)

$$(cq)^2 = 4E^2 \sin^2(\theta/2),$$

where E is the energy of the projectile, and θ the scattering angle.