UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2017/2018

TITLE OF PAPER: NUCLEAR PHYSICS

COURSE NUMBER: P442

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT TWO PAGES WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

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Useful Data:

1 unified mass unit $(u) = 1.6605 \times 10^{-27}$ kg $= 931.5$ MeV$/c^2$

Planck's constant $h = 6.63 \times 10^{-34}$ Js

Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K

Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1}

Speed of light (vacuum) $c = 3.0 \times 10^8$ m/s

electron mass $m_e = 9.11 \times 10^{-31}$ kg $= 5.4858 \times 10^{-4}$ u $= 0.511$ MeV$/c^2$

neutron mass $m_n = 1.6749 \times 10^{-27}$ kg $= 1.008665$ u $= 939.573$ MeV$/c^2$

proton mass $m_p = 1.6726 \times 10^{-27}$ kg $= 1.0072765$ u $= 938.280$ MeV$/c^2$

1year $= 3.156 \times 10^7$ s

nuclear radius, $R \approx r_0 A^{1/3}$, where $r_0 = 1.2$ fm

fine structure constant, $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}$

$\hbar c = 197$ MeVfm

The table of nuclear properties is provided in the next page.

Nuclide	Z	A	Atomic mass (u)	I^π	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	0^+	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8 s (β^-)
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.00000	0^+	99.89%
N	7	15	15.00109	$1/2^-$	0.366%
N	7	18	18.014081	1^-	0.63 s
O	8	15	15.003065	$1/2^-$	122 s
O	8	16	15.994915	0^+	99.76%
O	8	18	17.999160	0^+	0.204%
F	9	18	18.000937	1^+	110.0 min
Ne	10	20	19.992436	0^+	90.51%
Ne	10	22	21.991383	0^+	9.33%
Na	11	22	21.994434	3^+	2.60 yrs
Mg	12	21	21.000574	0^+	3.86 s
Al	13	27	26.981539	$5/2^+$	100.0%
Si	14	30	29.973770	0^+	3.10%
Si	14	32	31.974148	0^+	105 yrs
P	15	30	29.978307	1^+	2.50 min
P	15	32	31.971725	1^+	14.3 days
S	16	32	31.972071	0^+	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 days
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 days (β^-)
Sc	21	47	46.952409	$7/2^-$	3.35 days (β^-)
Fe	26	56	55.934439	0^+	91.8%
Fe	26	60	59.934078	0^+	1.5 Myrs
Co	27	60	59.933820	5^+	5.27 yrs
Ni	28	60	59.930788	0^+	26.1%
Ni	28	64	63.927968	0^+	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 hrs (β^-)
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	1^+	12.7 hrs
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	0^+	48.6%
Ru	44	104	103.905424	0^+	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44 hrs (β^-)
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 yrs (β^-)
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 days (β^-)
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%

Question 1

According to the liquid drop model the nuclear binding energy may be approximated by the semi-empirical formula

$$-B(A,Z) = -a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_A \frac{(Z-N)^2}{A} + \frac{[(-1)^Z + (-1)^N]}{2} \frac{a_P}{A^{1/2}},$$

where A is the atomic mass number, Z is the atomic number and N=AZ.

(a) Discuss the physical origin of each of the terms on the right-hand side of the above formula. (10)

(b) Considering a set of isobaric nuclei, show that the relationship between A and Z takes the form (10)

$$Z = \frac{A}{2 + \frac{a_c}{2a_A} A^{2/3}}$$

in the liquid drop model for naturally occurring nuclei with odd atomic mass number.

(c) State the assumptions of the liquid-drop model. (3)

(d) Why does the model fail to account for the binding energies of magic number nuclei? (2)

Question 2

The deuteron has total angular momentum $J = 1$ so that the ground state is a triplet state. The ground state is not a pure S state, i.e $L = 0$ state, and is formed by an admixture of the triplet S-state ($^3S_1$) and the triplet D(L=2)-state ($^3D_1$),

$$\Psi = \alpha\psi(^3S_1) + \beta\psi(^3D_1).$$

Here $|\alpha|^2 + |\beta|^2 = 1$.

(a) What is the value of the spin S for the deuteron? (2)

(b) Explain why the P-state, i.e $L = 1$ state is not compatible with the triplet S-state. (3)

(c) Given that the magnetic moment of deuteron is given by $\mu_D = \mu_p + \mu_n + \frac{1}{2}\mathbf{L} = g\mu_N\mathbf{J}$, show that (8)

$$g\mathbf{J} = \frac{1}{2}(g_p + g_n)\mathbf{S} + \frac{1}{2}(g_p - g_n)(\mathrm{s}_p - \mathrm{s}_n) + \frac{1}{2}\mathbf{L},$$

where $\mu_i = g_i\mu_N\mathrm{s}_i$, $\mathbf{S} = \mathrm{s}_p + \mathrm{s}_n$.

(d) Show that $\langle \mathrm{s}_p - \mathrm{s}_n \rangle = 0$ in triplet states. Hence show that (8)

$$g\mathbf{J} = \bar{g}\mathbf{S} + \frac{1}{2}\mathbf{L},$$

where $\bar{g} = \frac{1}{2}(g_p + g_n)$.

(e) Show that $g\mathbf{J}^2 = \bar{g}\mathbf{S}\cdot\mathbf{J} + \frac{1}{2}\mathbf{L}\cdot\mathbf{J}$. (4)

Question 3

(a) Summarize the standard model of elementary particles. (9)

(b) If the proton-electron model of the nucleus were valid, show that all neutral atoms would contain an even number of fermions. (6)

(c) Give two examples of each:

i. Quarks (2)

ii. Leptons (2)

iii. Baryons (2)

iv. Mesons (2)

v. Unifying Theories (2)

Question 4

(a) Consider a nuclide P with A^P total nucleons (Z^P protons and N^P neutrons)

i. write down the relation between nuclear mass, the proton mass, the neutron mass, and the binding energy, B_P^E. (2)

ii. In case of alpha decay $P \to D + \alpha$, derive the relation between the Q-value of the energy released and the binding energy of the parent nuclide B_P^E, the binding energy of the daughter nuclide, B_D^E, and the binding energy of the alpha -particle, B_α^E. (6)

(b) The beta decay process involves an emission of a neutrino as a third particle.

i. Explain how the energy spectrum of electrons produced in β-decay processes is evidence for the existence of the neutrino. (2)

ii. State the main properties ascribed to the neutrino. (3)

iii. Calculate the Q-value for neutron β-decay. (3)

(c) Briefly describe three processes through which gamma radiation interacts with matter. (9)

Question 5

(a) Write down an expression for the energy released during the spontaneous fission process of a nuclide. (2)

(b) Calculate the energy released in the process $^{235}\text{U} \rightarrow ^{140}\text{Xe} + ^{92}\text{Sr} + 3n$, given that the binding energies of the three nuclides are 1784 MeV for ^{235}U, 1161 MeV for ^{140}Xe and 796 MeV for ^{92}Sr. (3)

(c) Discuss qualitatively (with the aid of diagrams) how the shape of a nucleus changes during nuclear fission. (5)

(d) Hence explain why spontaneous fission is extremely rare, but fission can be induced by bombarding fissile material with neutrons. (4)

(e) Explain how a chain reaction occurs in nuclear fission. (3)

(f) Why is it necessary for a sample of fissile material to be greater than a critical mass in order for a chain reaction to occur? (4)

(g) In a nuclear reactor how is the chain reaction controlled in order to produce a steady output of energy? (4)