UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2017/2018

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P461

TIME ALLOWED : THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

APPENDICES 1 AND 2 CONTAIN DEFINITE INTEGRALS AND PHYSICAL CONSTANTS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

<u>Question One</u>

(a) (i) Explain briefly what is meant by *phase space*. (3 marks)
(ii) Define *density of states* in terms of energy ϵ. (2 marks)
(iii) Derive an expression for the volume element in phase space in terms of energy ϵ. and hence that of density of states. (6 marks)

(b) Explain the terms *macrostate* and *microstate* of a system of particles. (3 marks)

(c) Suppose we toss eight coins.
(i) What are the possible macrostates we can get? (2 marks)
(ii) Find the number of microstates corresponding to each macrostate. and hence the most probable macrostate? (5 marks)
(iii) What is the probability of getting this most probable macrostate? (2 marks)
(iv) What is the physical significance of *most probable macrostate* of a system of particles? (2 marks)

Question Two

(i) The partition function of a system Z is defined as $Z = \sum_s g_s e^{\beta \varepsilon_s}$. Derive the following expressions for the entropy and the total energy of a classical system:

(i) $$S = Nk \ln Z + \frac{E}{T}$$ (7 marks)

(ii) $$E = NkT^2 \frac{\partial}{\partial T} \ln Z$$ (6 marks)

(b) (i) Derive the partition function of a classical perfect gas,

$$Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$$

(See appendix for definite integrals) (5 marks)

(ii) Use the above result to show that the entropy of a classical gas:

$$S = Nk \ln\left[\frac{V}{h^3}(2\pi mkT)^{3/2}\right] + \frac{3}{2}Nk .$$ (5 marks)

Comment on the validity of this result. (2 marks)

Question Three

(a) (i) Show that for the most probable configuration the distribution of a system of bosons at temperature T can be represented as:

$$n_S = \frac{g_S}{e^{-(\alpha+\beta\varepsilon_S)} - 1}$$ where the symbols have their usual meanings

[Given that the weight of a system of bosons $W = \prod_S \frac{(g_S - 1 + n_S)!}{(g_S - 1)!n_S!}$]

(10 marks)

(ii) Show how at low density and at high temperature the above function approximates to the distribution function of a classical system

$$n_s = g_s \exp(\alpha + \beta\varepsilon_s)$$

[The multiplier α can be approximated to $\ln\frac{Nh^3}{V(2\pi mkT)^{3/2}}$] (4 marks)

(b) (i) State briefly what is meant by *Bose-Einstein condensation.* (3 marks)

(ii) Given that for this type of condensation $\frac{N'}{N} = \left(\frac{T}{T_B}\right)^{3/2}$, state what each symbol represents. (2 marks)

(iii) Find the relationship between the number of particles N_0 in the ground state and the temperature. (3 marks)

(iv) Calculate the condensation temperature of a system of bosons from the following data:
volume of one mole of the gas = 2.7×10^{-5} m^3
mass of a particle = 6.65×10^{-27} kg.

[Given $T_B = \frac{h^2}{2\pi mk}\left(\frac{N}{2.612V}\right)^{2/3}$] (3 marks)

Question Four

(a) Given that a one-dimensional harmonic oscillator has discrete energy given by $\varepsilon = \left(n + \frac{1}{2}\right)h\nu$, where symbols have their usual meanings, obtain an expression for its mean energy. (9 marks)

$$\text{[Given: mean energy } = kT^2\left(\frac{\partial \ln Z}{\partial T}\right)_V \text{]}$$

(b) Assume that a solid has N atoms each having three mutually independent vibrations. Using your results in question (a) above, obtain an expression to show how the specific heat capacity of the solid varies with temperature.. (8 marks)

(c) Show that at high temperatures specific heat capacity of the solid is equal to the classical value $3Nk$. (4 marks)

(d) Experimentally it is observed that at low temperatures specific heat of solids varies proportional to T^3. Does the above theory agree with this? Comment. (4 marks)

Question Five

(a) The Fermi function of a system is given as: $f(\varepsilon) = \frac{1}{e^{(\varepsilon-\varepsilon_F)/kT}+1}$ where symbols have their usual meanings. Obtain its values at absolute zero, for the cases

(i) $\epsilon > \epsilon_F$ and (2 marks)

(ii) $\epsilon < \epsilon_F$ (2 marks)

(iii) What is the physical meaning of these results? (2 marks)

(b) The Fermi energy of a solid is 8.6 eV. Find the probability of occupation of an electron

(i) in an energy level 0.1 eV above the Fermi level at 300 K and at 400 K. (2 marks)

(ii) in an energy level 1.0 eV above the Fermi level at 300 K and at 400 K. (2 marks)

(iii) Comment on the results. (2 marks)

(c) Derive an expression for the paramagnetic susceptibility of a metal to show that it is independent of temperature. Draw diagrams where necessary.

[Neglect the response to the applied field due to the orbital motion of the electrons. Assume energy $\mu_B B \ll \epsilon_F$ and ϵ_F is a constant for the material]

Given the density of states per unit volume $g(\varepsilon)d\varepsilon = \frac{4\pi}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon.$

(13 marks)

Appendix 1

Various definite integrals

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty e^{-ax}dx = \frac{1}{a},(a > 0)$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Planck's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23} mol^{-1}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8} $Wm^{-2}K^{-4}$
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of ${}_2^4$He atom		6.65×10^{-27} kg
Mass of ${}_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 L mol^{-1}