UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2017 /2018

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P461

TIME ALLOWED : THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

APPENDICES 1 AND 2 CONTAIN DEFINITE INTEGRALS AND PHYSICAL CONSTANTS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One

(a) (i) Explain what is meant by *phase space.* (4 marks)

(ii) What is the volume of a state in phase space? Show that your answer is dimensionally correct. (1+ 3 marks)

(iii) What is the number of allowed states per unit volume in phase space? (1 mark)

(b) Given that the density of states $g(\varepsilon)d\varepsilon = \frac{2\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$, show that the Maxwell-Boltzmann distribution function can be written in the differential form

$$n(v)dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/(2kT)}v^2 dv.$$

(5 marks)

(c) (i) State *the law of equipartition of energy.* . (3 marks)

(ii) Prove the above law for the case in which the energy is a quadratic function of position.

(8 marks)

Question Two

(a) Derive the partition function of a classical gas:

$$Z = \frac{v}{h^3}(2\pi mkT)^{3/2}$$ (8 marks)

(b) (i) Show that the pressure of the classical gas $P = NkT\frac{\partial \ln Z}{\partial V}$. (5 marks)

(ii) Hence derive the ideal gas equation $P\,V = N\,k\,T$ (5 marks)

(c) Calculate the translational partition function of an hydrogen molecule confined to a volume of 100 cm^3 at 300 K . (7 marks)

Question Three

(a) Write down the Bose-Einstein distribution function for a system of bosons and show under what conditions it can approximate the classical Maxwell-Boltzmann distribution function. (7 marks)

$$\text{Given: } \alpha = \ln\left[\frac{Nh^3}{(2\pi mkT)^{3/2}V}\right]$$

(b) Use the Bose-Einstein distribution function of an assembly of identical non-interacting particles in thermal equilibrium to derive the Planck's radiation law for spectral distribution of energy radiated from a constant temperature enclosure. (11 marks)

(c) Obtain an expression for the total energy per unit volume emitted from the enclosure at temperature T. (7 marks)

Question Four

(a) State what is meant by *Fermi energy*. (2 marks)

(b) Show that the average kinetic energy of a particle in a Fermi gas at 0 K is (3/2) times the Fermi energy at 0 K , written as ϵ_F (0).

Density of states of fermions $g(\varepsilon)d\varepsilon = \frac{4\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$

Average energy $E_{ave} = \frac{1}{N}\int \varepsilon dN$

(5 marks)

(c) (i) By deriving an appropriate expression, show that the contribution of electrons towards the heat capacity of a material is proportional to its temperature. (10 marks)

(ii) In sodium, there are about 2.6 x10^{28} electrons per cubic meter. Calculate the value of the heat capacity per electron of sodium at 300 K. (8 marks)

$$N = \frac{8\pi V(2m)^{3/2}}{3h^3}\varepsilon_F^{3/2}$$

Question Five

(a) Derive the Fermi-Dirac distribution function for a system of fermions,

$$n_S = \frac{g_S}{e^{-(\alpha+\beta\varepsilon_S)} + 1}$$, where the symbols have their usual meanings.

(12 marks)

(b) (i) Given that the density of states of a system of fermions is:

$$g(\varepsilon)d\varepsilon = \frac{4\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$$

where the symbols have their usual meanings, show that the Fermi energy of a system of fermions

$$\varepsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

(8 marks)

(ii) Calculate the Fermi energy of a metal having density 8.5 x10^2 kg m^{-3} and atomic weight 40. (5 marks)

Appendix 1

Various definite integrals

$$\int_0^\infty e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\, x\, dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2} x^3\, dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\, x^2\, dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\, x^4\, dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\, x^5\, dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\, dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty e^{-ax} dx = \frac{1}{a}, (a > 0)$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2}\, dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\, e^{-\lambda x}\, dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

Appendix 2

Physical Constants

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Planck's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23} mol^{-1}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8} Wm^{-2}K^{-4}
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of $_2^4$He atom		6.65×10^{-27} kg
Mass of $_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 L mol^{-1}