UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2017/2018

TITLE OF PAPER: MECHANICS

COURSE NUMBER: PHY211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS.

EACH QUESTION CARRIES 25 MARKS.

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN.

THIS PAPER HAS 6 PAGES INCLUDING THE COVER PAGE.

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QUESTION 1

(a) Derive the basic kinematic equation:

$$v^2 = v_0^2 + 2a_0(x - x_0).$$

(5 marks)

(b) A person throws a stone at an initial angle $\theta = 90°$ from the horizontal with an initial speed of $v_0 = 20\text{m/s}$. The point of release of the stone is at a height $d = 20\text{m}$ above the ground. Neglect air resistance. The gravitational acceleration $g = 9.81\text{m/s}$. Determine:

(i) The maximum height, y_{max} reached by the particle and the time at y_{max}.

(7 marks)

(ii) The time the particle reaches the ground.

(3 marks)

(c) A person is standing on a ladder holding a bucket. The person releases the bucket from rest at a height h_1 above the ground. A second person standing a horizontal distance s_2 from the bucket aims and throws a ball the instant the bucket is released in order to hit the bucket. The person releases the ball at a height h_2 above the ground, with an initial speed v_0 , and at an angle θ_0 with respect to the horizontal. Ignore air resistance.

(i) Find the position of the bucket as a function of time.

(3 marks)

(ii) Find the position of the ball as a function of time.

(3 marks)

(iii) Find an expression for the angle θ_0 that the person aims the ball in order to hit the bucket.

(4 marks)

QUESTION 2

(a) A sketch of a "pedagogical machine" is shown below. All surfaces are frictionless. A force F is applied to M_1 to keep M_3 from rising or falling?

(i) Draw force diagrams for the system.

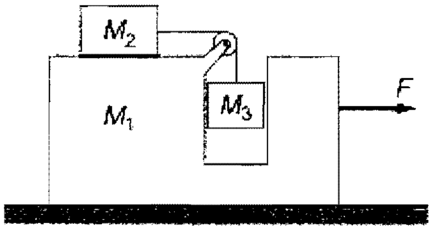
(5 marks)

(ii) Find the tension T in the string.

(3 marks)

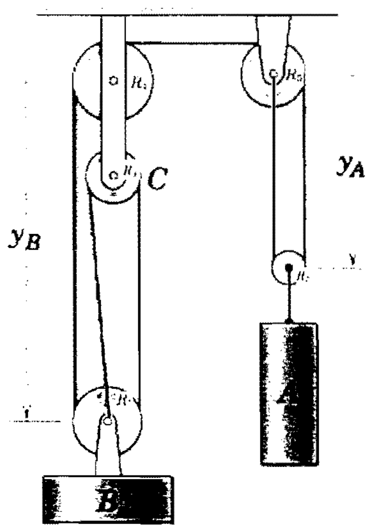
(iii) Find the force F that is applied to M_1 to keep M_3 from rising or falling.

(10 marks)



(b) Consider the pulley shown below, determine the velocity of B if A has a downwards velocity of $v = 0.6\text{m/s}$.

(7 marks)



QUESTION 3

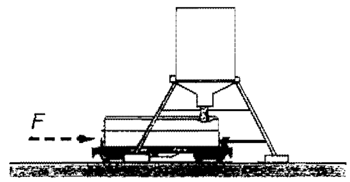
(a) An empty rail car of mass M starts from rest under an applied force F. At the same time, sand begins to run into the car at steady rate b from a hopper at rest along the track.

(i) Find the velocity when a mass of sand m has been transferred to the rail car. The problem can be solved in only two steps, but use the *mass and momentum transport* method.

(8 marks)

(ii) Apply your solution to the case when $M_0 = 400$ kg, $b = 15$ kg/s and $F = 80$ N to find the velocity at time $t = 10$ s.

(2 marks)



(b) A cylindrical rocket of diameter $2R$, mass M_R and containing fuel of mass M_F is coasting through empty space at velocity v_0. At some point the rocket enters a uniform cloud of interstellar particles with number density N with each particle having mass m ($m \ll M$) and initially at rest. To compensate for the dissipative force of the particles colliding with the rocket, the rocket engines emits fuel at a rate $dm/dt = \gamma$ at a constant velocity u with respect to the rocket. Neglect gravitational effects between the rocket and cloud particles.

(i) Assuming that the dissipative force from the cloud particles takes the form $F = -Av^2$, where A is a constant, derive the equation of motion of the rocket through the cloud as it is firing its engines.

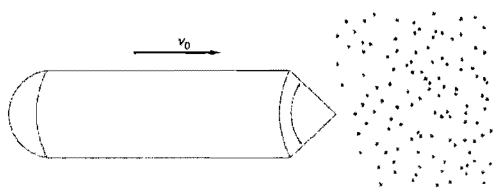
(5 marks)

(ii) What must the rocket's force be to maintain a constant velocity v_0?

(5 marks)

(iii) If the rocket suddenly runs out of fuel, what is its velocity as a function of time after this point?

(5 marks)

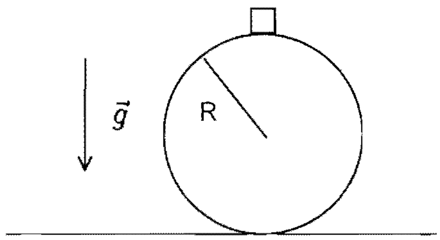


QUESTION 4

(a) Derive an expression for the work-energy theorem?

(8 marks)

(b) A small block starts from rest and slides down the top of a fixed sphere of radius R, where $R \gg$ size of the block as shown below. The surface of the sphere is frictionless and constant gravitation acceleration g acts downwards.



(i) Determine the speed of the block as a function of angle from the top while it remains in contact with the sphere.

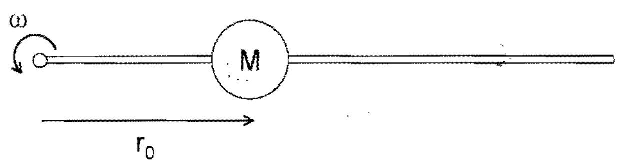
(8 marks)

(ii) At what angle does the block lose contact with the sphere?

(9 marks)

QUESTION 5

(a) A bead of mass M is placed on a frictionless, rigid rod that is spun about at one end at a rate ω. The bead is initially held at a distance r_0 from the end of the wire. Treat the bead as a point mass and neglect gravitational forces.



(i) What force is necessary to hold the bead in place at r_0?

(5 marks)

(ii) After the bead is released, what is its position in the inertial frame (in polar coordinates) as a function of time?

(8 marks)

(b) A small planet of mass m is in a circular orbit of radius r around a star of mass M in otherwise empty space (assume $M \gg m$ so the star is stationary). Assuming $U \to 0$ as $r \to 0$ determine in terms of G, M and r:

(i) The potential energy $U(r)$ of the planet.

(6 marks)

(ii) The kinetic energy $K(r)$ of the planet.

(6 marks)