UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2017/2018

TITLE OF PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: PHY221/P221

TIME ALLOWED: THREE HOURS

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INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE WHEN NECESSARY.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Useful Mathematical Relations

**Fundamental Theorem of Gradients**

$$\int_{\vec{a}}^{\vec{b}} (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$$

**Divergence Theorem**

$$\int \nabla \cdot \vec{A} d\tau = \oint \vec{A} \cdot d\vec{a}$$

**Curl Theorem**

$$\int (\nabla \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

**Line and Volume Elements**

Cartesian: $d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$, $d\tau = dxdydz$

Cylindrical: $d\vec{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$, $d\tau = sdsd\phi dz$

Spherical: $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$, $d\tau = r^2 \sin\theta dr d\theta d\phi$

**Gradient and Divergence in Spherical Coordinates**

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

**Dirac Delta Function**

$$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$$

**Question 1: ELECTROSTATICS**..............................................

(a) Consider a linear charge distribution along the x-axis, which starts from $x = -a$ and runs up to $x = +a$, with uniform density $+\lambda$.

i. To simplify the analysis, first consider a point charge at some arbitrary position $x$ such that $x < a$, along the x-axis. What is the electric field at some position $L$ along the x-axis such that $L > a$? (2)

ii. Use the result above (the point charge case) to determine the electric field at position $L$ along the x-axis, such that $L > a$, for the linear charge distribution. (6)

iii. What would be the force on a point charge of charge $+q$ placed at position $L$ on the x-axis? (2)

iv. If the charge $+q$ is originally at $x = \infty$, calculate the work done in bringing it to the position $L$. (6)

(b) Starting from the fact that an electrostatic field is *irrotational*, show that the line integral $\int_{\vec{a}}^{\vec{b}} \mathbf{E} \cdot d\mathbf{l}$ is path independent. (4)

(c) The path independence of the line integral $\int_{\vec{a}}^{\vec{b}} \mathbf{E} \cdot d\mathbf{l}$ allows for the definition of the electric potential from the electric field. Use the potential difference between two positions **a** and **b** and the fundamental theorem of gradients to show that the electric field can be expressed as the negative gradient of the electric potential, i.e $\mathbf{E} = -\vec{\nabla} V$. (5)

**Question 3: Magnetostatics** ........................................................

(a) Consider a solenoid of length $l$ and radius $a$ containing $N$ closely spaced turns and carrying a steady current $I$.

i. In terms of these parameters, find the magnetic field at a point along the axis as a function of position $x$ from the end of the solenoid. (**hint**: Consider the field due to a circular loop first.) (10)

ii. Show that as $l$ becomes very long, $B$ approaches $\mu_0 NI/2l$ at each end of the solenoid. (2)

(b) A wire is formed into the shape of a square of edge length $L$. Assume the sqaure is placed on the xy plane with its center at the origin.

i. Show, using the Biot-Sarvat law, that when the current in the loop is $I$, the magnetic field at a point a distance $z$ above the center of the square, i.e the field point is on the z-axis, is (8)

$$B = \frac{\mu_0 I L^2}{2\pi(z^2 + L^2/4)\sqrt{z^2 + L^2/2}}$$

ii. Show that the field reduces to $2\sqrt{2}\mu_0 I/\pi L$ at the origin. (2)

(c) Explain why two parallel wires carrying currents in opposite directions repel each other. (3)

**Question 4: Magnetostatics II** ..........................................

(a) State Maxwell's equations for time independent fields. (4)

(b) A certain magnetic field has the form $\mathbf{B} = (ax/y^2)\hat{x} + (by/x^2)\hat{y} + f(x, y, z)\hat{z}$, where $a$ and $b$ are constants.

i. Find the most general form for the function $f(x, y, z)$. (5)

ii. Find the current density $\mathbf{J}$. (5)

iii. Verify, using the continuity equation, that the current density corresponds to a steady current distribution. (3)

(c) Describe the mechanism responsible for paramagnetism. (4)

(d) Describe the mechanism responsible for diamagnetism. (4)

Question 5: **Electrodynamics and Alternating Current Circuits** . . . . . . . . . . .

(a) Can a battery be used as a primary voltage source in a transformer? Explain your answer. (3)

(b) A capacitor with capacitance $C$ is charged up to a potential $V$ and connected to an inductor with inductance $L$. At time $t = 0$ a switch is closed.

i. Find the current in the circuit as a function of time. (8)

ii. How does the answer change if a resistor $R$ is included in series with $C$ and $L$? (10)

(c) When a resistor is connected to a voltage source as the only load, there is no phase difference between voltage and current.

i. What is the phase difference in a circuit with a purely capacitive load? (2)

ii. What is the phase difference in a circuit with a purely inductive load? (2)

**Note**: To solve an ordinary differential equation of the form

$$\frac{d^2y}{dt^2} + ay = 0,$$

assume the solution is of the form $y = y_0 \exp(-bt)$, then find $y_0$ and $b$.