UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

RE-SIT/SUPPLEMENTARY EXAMINATION: 2017/2018

TITLE OF PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: PHY221/P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE WHEN NECESSARY.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Useful Mathematical Relations

Gradient Theorem

$$\int_{\vec{a}}^{\vec{b}} (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$$

Divergence Theorem

$$\int \nabla \cdot \vec{A} d\tau = \oint \vec{A} \cdot d\vec{a}$$

Curl Theorem

$$\int (\nabla \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

Line and Volume Elements

Cartesian: $d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$, $d\tau = dxdydz$

Cylindrical: $d\vec{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$, $d\tau = sdsd\phi dz$

Spherical: $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$, $d\tau = r^2 \sin\theta dr d\theta d\phi$

Gradient and Divergence in Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Dirac Delta Function

$$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$$

Question 1: Electrostatics ....

(a) Two point charges attract each other with an electric force of magnitude F. What is the resulting magnitude of the force (in terms of F) in the following cases

i. The charge on one of the particles is reduced to one third its original value and the distance between the particles is doubled. (2)

ii. The charge on both particles is doubled and the distance between them is reduced to half the original value. (2)

iii. The charge on one particle is reduced to one quarter its original value and the distance between the particles is reduced to one half its original value. (2)

(b) Consider a charge Q distributed through a sphere of radius R with a charge density $\rho = A(R - r)$ (in spherical coordinates). Here ρ is volume charge density.

i. Determine the constant A in terms of Q and R. (4)

ii. Hence deduce the SI unit for A. (1)

iii. Use Gauss' law to determine the field at an arbitrary point inside the sphere. (8)

iv. Use Gauss' law to determine the field at an arbitrary point outside the sphere. (6)

Question 2: Electrostatic II ....

(a) Write short notes on polarization in dielectrics by describing the two effects of external electrostatic fields on such materials. (6)

(b) Prove that if an empty cavity is surrounded by a conductor, $\vec{E} = 0$ inside the cavity regardless of the presence of fields external to the conductor. (5)

(c) Consider a square with sides of length a and three positive charges, q at each of three corners. To bring in another positive charge from infinity and place it on one of the corners requires work to be done. Show that the amount of work done to bring the charge is $W = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a}(2 + 1/\sqrt{2})$. (8)

(d) Use the divergence theorem together with the integral version of Gauss' law to derive the differential form of Gauss' law. (6)

Question 3: Magneto-statics..

(a) A circular loop of radius a lies in the xy plane with the origin at its center.

i. Use the Biot-Sarvat law to find the magnetic field at any point a distance $z > 0$ above the center of the square, i.e along the z-axis, when a current **I** circulates counter clockwise around the square. (8)

ii. Show that $B = \mu_0 I/2a$ at the center of the square, i.e at the origin. (2)

(b) Show that if a charge moves a distance $d\vec{l}$ the work done by magnetic forces is zero. (5)

(c) A square of edge $2a$ lies in the xy plane with the origin at the center. The sides of the square are parallel to the axes, and a current I goes around it in a counterclockwise sense as seen from a positive value of z.

i. Find **A** at the origin. (8)

ii. What is the $\nabla \cdot \mathbf{A}$ at the origin? (2)

Note:

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln |x + \sqrt{a^2 + x^2}| + C$$

Question 4: Magnetostatcs II ....

(a) Derive the continuity equation (8)

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t},$$

starting from the definition $\mathbf{J} = \frac{d\mathbf{I}}{da_{\perp}}$

(b) Suppose a current I is uniformly distributed over a wire of circular cross section with radius a.

i. Determine the volume current density $\mathbf{J}$. (4)

ii. How does the corresponding charge density depend on time? (3)

(c) Use Ampere's law to find the magnetic field a distance s from a long straight wire, carrying a steady current I. (5)

(d) Use Ampere's law to find the magnetic field of a very long solenoid, consisting of n closely wound turns per unit length on a cylinder of radius a. (5)

Question 5: Circuits and Electrodynamics ....

Imagine charging up a capacitor by connecting it to a battery and a resistor, with the battery's voltage fixed at V_0.

(a) Determine the charge $Q(t)$ and current $I(t)$ as functions of time. (8)

(b) Find the total energy of the battery. (5)

(c) Determine the heat delivered to the resistor. (5)

(d) What is the final energy stored in the capacitor? (5)

(e) What fraction of the work done by the battery shows up as energy in the capacitor? (2)