UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2017/2018

TITLE OF PAPER:	MODERN PHYSICS AND WAVE OPTICS
COURSE NUMBER:	PHY232
TIME ALLOWED:	THREE HOURS
INSTRUCTIONS:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS.
	EACH QUESTION CARRIES 25 MARKS.
	MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN.

THIS PAPER HAS 6 PAGES INCLUDING THE COVER PAGE.

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QUESTION 1

(a) Consider two coherent light waves of the same angular frequency ω and wave number k moving along the positive x axis towards a screen, and described by the following equations:

$$y_1 = A\sin(kx - \omega t)$$
$$y_2 = A\sin(kx - \omega t + \phi)$$

Find the sum of the two waves $y = y_1 + y_2$, and describe the wave formed.

(6 marks)

(b) Two narrow, parallel slits separated by $d = 0.250$ mm are illuminated by green light ($\lambda = 546.1$nm). The interference pattern is observed on a screen 1.20 m away from the plane of the parallel slits.

(i) Calculate the distance from the central maximum to the first bright region on either side of the central maximum.

(4 marks)

(ii) Calculate the distance between the first and second dark bands in the interference pattern.

(5 marks)

(c) Two slits are separated by 0.180 mm. An interference pattern is formed on a screen 80.0 cm away by light with $\lambda = 656.3$nm. Calculate the fraction of the maximum intensity a distance $y = 50.600$ cm away from the central maximum.

(4 marks)

(d) A soap bubble ($n = 1.33$) floating in air has the shape of a spherical shell with a wall thickness of 120 nm.

(i) Make a diagram that you can use as a basis to solve this problem.

(3 marks)

(ii) What is the wavelength of the visible light that is most strongly reflected?

(3 marks)

QUESTION 2

(a) A helium-neon laser emits light that has a wavelength of 632.8 nm. The circular aperture through which the beam emerges has a diameter of 0.500 cm. Estimate the diameter of the beam 10.0 km from the laser.

(6 marks)

(b) A diffraction grating has 4200 rulings/cm. On a screen 2.00 m from the grating, it is found that for a particular order m, the maxima corresponding to two closely spaced wavelengths of sodium (589.0 nm and 589.6 nm) are separated by 1.54 mm. (Do **not** use small angle approximation, $\sin\theta \approx tan\theta$ in this case.)

(i) What is the grating spacing d?

(2 marks)

(iii) At what angle do each of the two lines appear in the fourth order spectrum?

(5 marks)

(ii) Determine the value of m, where the maxima corresponding to two closely spaced wavelengths of sodium (589.0 nm and 589.6 nm) are separated by 1.54 mm.

(8 marks)

(c) The critical angle for total internal reflection for sapphire surrounded by air is 34.4°. Calculate the polarizing angle for sapphire.

(4 marks)

QUESTION 3

(a) Assuming the human body and a tungsten filament both emit like blackbodies,

(i) Calculate the peak wavelengths of the human skin at a temperature of 310 K, and a tungsten incandescent light bulb at a temperatur of 3 000 K.

(4 marks)

(ii) Calculate the intensity $I(\lambda, T)$ per unit area, at the given temperatures given in part 2.(a)(i) for the peak sensitivity of the human eye $\lambda = 550$nm.

(3 marks)

(iii) Explain why a person is able to clearly see the light of the tungsten light bulb but not that of the human being on a dark night?

(3 marks)

(b) A lithium surface is illuminated with light, the work function for lithium metal is $\phi = 2.93eV$.

(i) What frequency of light is needed to produce electrons of kinetic energy 3.00 eV through this process?

(4 marks)

(ii) Find the cutoff wavelength λ_c for lithium.

(3 marks)

(c) An x ray of wavelength 0.050 nm scatters from a gold target.

(i) Can the x ray be Compton-scattered from an electron bound by as much as 62 keV?

(4 marks)

(ii) What is the largest wavelength of scattered photon that can be observed?

(4 marks)

QUESTION 4

(a) A photon is emitted when a hydrogen atom undergoes a transition from the $n = 5$ state to the $n = 3$ state.

(i) Calculate the energy (in electron volts) of the emitted photon.

(3 marks)

(ii) For the emitted photon calculate the frequency.

(3 marks)

(iii) Calculate the wavelength of the emitted photon.

(3 marks)

(iv) Calculate the wavelength using the empirical Balmer-Rydberg equation and compare to the results from the Bohr model.

(3 marks)

(b) A hydrogen atom is in its second excited state, corresponding to $n = 3$. Find:

(i) The radius of the electron's Bohr orbit.

(3 marks)

(ii) The de-Broglie wavelength of the electron in this orbit.

(4 marks)

(c) What are the three primary properties of laser light?

(6 marks)

QUESTION 5

(a) Starting with the equation for the decay rate of N nuclei

$$\frac{dN}{dt} = -\lambda N,$$

where λ is the decay constant, show that the half-life of a radioactive substance is given by

$$T_{1/2} = \frac{\ln 2}{\lambda}.$$

(10 marks)

(b) At time $t = 0$, a radioactive sample contains $3.50\mu g$ of pure ${}^{11}_{6}C$, which has a half-life of 20.4 minutes.

(i) Determine the number N_0 of nuclei in the sample at $t = 0$.

(5 marks)

(ii) What is the activity of the sample initially and after 5.00 h?

(5 marks)

(c) Find the energy released in the deuterium-deuterium reaction

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H$$

(5 marks)

SOME INFORMATION THAT MAY BE USEFUL IN SOME PROBLEMS

Avogadro's number $A = 6.02 \times 10^{23}$ particles per mole
Bohr radius $a_0 = 5.291\,772 \times 10^{-11}$ m
Boltzmann's constant, $k_B = 1.3801 \times 10^{-23}$ J/K
Compton wavelength $\lambda_C = \frac{h}{m_e c} = 0.002\,43$ nm
Coulomb constant $k_e = 8.987\,551\,788 \times 10^{23}9$ N.m^2/C^2
Radii of orbit for the hydrogen atom $r_n = n^2 a_0$
Rydberg constant $R_H = 1.097\,373 \times 10^7$ m^{-1}.
Planck's constant, $h = 6.626\,075 \times 10^{-34}$ Js
$\hbar = 1.054\,572 \times 10^{-34}$ Js
$hc = 1.986\,447 \times 10^{-25}$ Jm
$2\pi hc^2 = 3.741\,859 \times 10^{-15}$ J·m^2·s^{-1}
Speed of light in vacuum, $c = 2.997\,924\,58 \times 10^8$ m/s
Stefan-Boltzmann Constant $\sigma = 335.669\,6 \times 10^{-8}$ W/(m^{2}K^2)
Wien's displacement law $\lambda_{max} = \frac{hc}{4.965 k_B T}$

Nuclear Data

Electron charge, $e = 1.602\,177\,33 \times 10^{-19}$ C
Electron mass, $m_e = 9.109\,389\,7 \times 10^{-31}$ kg $= 0.000\,548\,6$ u
Neutron, $m_n = 1.674\,928\,6 \times 10^{-27}$ kg $= 1.008\,665$ u
Proton mass, $m_p = 1.672\,623 \times 10^{-27}$ kg $= 1.007\,276$ u

1 atomic mass unit = 1 amu = 1 u = $1.660\,540\,2 \times 10^{-27}$ kg $\equiv$ 931.494 MeV rest mass energy
1 eV = $1.602\,177\,33 \times 10^{-19}$ J: 1 MeV = $1.602\,177\,33 \times 10^{-13}$ J
$T_{1/2}(^{14}C) = 5730$ years
$T_{1/2}(^{238}U) = 4.47 \times 10^9$ years
Ratio of carbon 14 to carbon 12 in the atmosphere $= \frac{N(^{14}C)}{N(^{12}C)} = 1.2987 \times 10^{-12}$
1Curie (Ci) = 3.7×10^{10} Becquerel (Bq)
$r_0 = 1.2 \times 10^{-15}$ m
Alpha particle (α) (^{4}He) atomic mass = 4.002 603 u
Cerium ($^{140}_{58}$Ce) atomic mass = 139.905 434 u
Deuterium (^{2}D) atomic mass = 2.014 102 u
Hydrogen (^{1}H) atomic mass = 1.007 825 u
Hydrogen molecular mass = 1.007 94 u
Iron (^{56}Fe) atomic mass = 55.934 942 u
Molybdenum (^{94}Mo) mass = 93.905 088 u
Neodymium (^{144}Nd) atomic mass = 143.910 083 u
Oxygen ^{15}O atomic mass = 15 .003 065 u
Nitrogen ^{15}N atomic mass = 15.000 109 u
Radium ($^{226}_{88}Ra$) atomic mass = 226.025 403 u
Radon ($^{222}_{86}Rn$) atomic mass = 222.017 570 u
Ruthenium (^{98}Ru) atomic mass = 97.905 287 u
Tritium (^{3}T) atomic mass = 3.016 049 u

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$