UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2017/2018

TITLE OF THE PAPER: COMPUTATIONAL PHYSICS I

COURSE NUMBER: PHY282

TIME ALLOWED:
SECTION A: ONE HOUR
SECTION B: TWO HOURS

INSTRUCTIONS:
THERE ARE TWO SECTIONS IN THIS PAPER:

- **SECTION A** IS A WRITTEN PART. ANSWER THIS SECTION ON THE ANSWER BOOK. IT CARRIES A TOTAL OF **40** MARKS.
- **SECTION B** IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. IT CARRIES A TOTAL OF **60** MARKS.

Answer **all** the questions from Section A and **all** the questions from Section B. Marks for different sections of each question are shown in the right hand margin.

THE PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

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Section A – Use a pen and paper to answer these questions

Question 1

(a) In simple words, what is an interpreted programming language?

[2 marks]

(b) State two tasks that can be preferably done in MAPLE compared to complied languages such as Fortran or C/C++.

[2 marks]

(c) Explain the difference between the following Maple input statements and functions:

(i) `>eq:=2*I+10;` and `>eq:=2*I+10:`

(ii) `> sqrt(12);` and `>evalf(sqrt(12),20);`

(iii) `dsolve()` and `fsolve()`

[6 marks]

Question 2

(a) What is meant by a differential equation and give an example of a law in physics that can be represented by a differential equation?

[2 marks]

(b) Are the following equations linear or nonlinear?

(i) $d^4\theta(t)/dt^4 + \tau\theta(t) = 0$

(ii) $\dot{y}(t) = 1/y(t)$

(iii) $\ddot{x}(t) = -\omega_0^2 x(t) + 0.5\cos(\omega t)$

(iv) $\dot{P}(t) = \alpha P(t)(1 - P(t)/K)$

[4 marks]

(c) The motion of a particle is described by the following differential equation

$$\ddot{x} + \mu\dot{x} + x = 0$$

where $x(t)$ is the displacement and μ is a positive dimensionless coefficient of nonlinear friction. Discuss how you would decompose this equation a system of first order differential equations? How many initial value conditions are required to solve this equation.

[4 marks]

Question 3

(a) What is the value of C after the following statements have been executed?

```
>with(LinearAlgebra);
>A:=Matrix([[5,6,7,8],[9,10,11,112],[13,14,15,16],[17,18,19,20]]);
>B:=Matrix(4,4,[3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3])
>C:=A+B;
```

[2 marks]

(b) What values of x and y are given out after the following statements have been executed?

```
x:=4;y:=-2;
s:=x+y;
x:=x+x/s;
s:=x+y;
y:=y+x/s;
```

[2 marks]

(c) Translate the following expressions into Maple input statements

(i) $y^2 - \frac{3}{\cos(\phi)} + \frac{7}{\cos^2(\phi)} - 12$

(ii) $1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \cdots + \frac{1}{100^3}$

(iii) $g(x) = \begin{cases} x^2 & x \le 0 \\ \cos(x) & 0 < x \le 2\pi \\ e^{-x} & 2\pi < x \end{cases}$

[6 marks]

Question 4

(a) The program below is supposed to convert the temperature of boiling water (212 °F) from Fahrenheit to Celsius (°C) using the conversion formula

$$C = \frac{5}{9}(f - 32)$$

but it does not produce the correct result. Discuss briefly what is the output of the program. Fix it such that the originally intended purpose is restored.

```
Fer_2_Cel:=proc(x)
C:=5/9*f-32;
return C end proc:
Fer_2_Cel(212);
```

[4 marks]

(b) Describe exactly but briefly what is the output of the program:

```
> programX:=proc(a,b,c)
local r1,r2;
r1:=(-b+sqrt(b**2-4*a*c))/(2*a)
r2:=(-b-sqrt(b**2-4*a*c))/(2*a)
return r1,r2;
end proc;
> programX(1,2,3);
```

[3 marks]

(c) The function below is supposed to return the sum $\sum_{i=1}^{N}(i^2+1)$ given a positive integer N but it does not. Fix it such that the originally intended purpose is restored.

```
SuM:=proc(N)
x:=0;
for i from 0 to N do
x:=i*2+1;
end do;
end proc;
```

[3 marks]

Section B – Practical Part

Question 5

(a) The Maple program below is supposed to calculate the range of an ideal projectile given the launch angle θ (in degrees) and the initial speed $v0$ using the formula

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

but it does not produce the correct result. Fix it such that the originally intended purpose is restored and plot `Range(T,15)` for $T = 0 \ldots 80°$

```
Range:=proc(T,v0)
local R, theta,g;
theta:=T/180*Pi;
R:=v0*2* sin(2)*theta/g;
end proc:
```

[10 marks]

(b) Consider an oscillatory potential

$$V(r) = a\cos(r) + b\cos(2r) - cr$$

with $a = 1.5, b = 2$, and $c = 0.5$.

(i) Plot $V(r)$ for $r = -12 \ldots 12$.

[5 marks]

(ii) Find the local maximum and the local minimum of $V(r)$ close to $r = 0$.

[5 marks]

Question 6

(a) Applying Kirchhoff's law to an electrical network leads to the following systems of 5 linear equations

$$\begin{aligned} 1.5i_1 - 2i_2 + i_3 + 3i_4 + 0.5i_5 &= 7.5 \\ 3i_1 + i_2 - i_3 + 4i_4 - 3i_5 &= 16 \\ 2i_1 + 6i_2 - 3i_3 - i_4 + 3i_5 &= 78 \\ 5i_1 + 2i_2 + 4i_3 - 2i_4 + 6i_5 &= 71 \\ -3i_1 + 3i_2 + 2i_3 + 5i_4 + 4i_5 &= 54 \end{aligned}$$

where i_1, i_2, i_3,i_4, and i_5 are currents in Amperes. Determine these unknown values.

[10 marks]

(b) Consider the function

$$S(N) = \sum_{x=1}^{N} \frac{1}{x} - ln(N).$$

where N is a positive integer. Write a Maple procedure $S(N)$ that define this function. Plot $S(N)$ for N =10...1000 and show that for large N, this function converges to the value of the Euler constant $\gamma = 0.577215664901532\ldots$

[10 marks]

Question 7

A set of magnetically-coupled rotors with a velocity dependent damping move according to the equations:

$$\begin{aligned}\ddot{\phi}_1(t) &= -a\sin(\phi_2(t) - \phi_1(t)) - b\dot{\phi}_1(t)\\ \ddot{\phi}_2(t) &= -a\sin(\phi_1(t) - \phi_2(t)) - b\dot{\phi}_2(t).\end{aligned} \tag{1}$$

This system exhibits an intriguing oscillatory behavior, where the two rotors alternatively exchange angular velocity.

(a) Write a program that determines $\phi_1(t)$, $\phi_2(t)$, $\dot{\phi}_1(t)$ and $\dot{\phi}_2(t)$ for a system with the parameters $a = 5.0$, $b = 0.2$ and the initial conditions

$$\phi_1(0) = 0, \quad \phi_2(0) = 1, \quad \dot{\phi}_1(0) = 0, \quad \text{and} \quad \dot{\phi_2(0)} = 0.$$

NB: *You may need to decompose the above equation into a system of two first order ODEs.*

[10 marks]

(b) Plot $\phi_1(t)$, $\phi_2(t)$ versus time t on the same plot for $t = 0\ldots 20$.

[4 marks]

(c) Plot $\dot{\phi}_1(t)$, $\dot{\phi}_2(t)$ versus the time t on the same plot for $t = 0\ldots 20$ and confirm that the two rotors alternatively exchange angular velocity.

[4 marks]

(d) Is the total angular momentum conserved or not in this system?

[2 marks]