UNIVESITY OF SWAZILAND
FACULTY OF SCIENCE AND ENGINEERING
DEPARTMENT OF PHYSICS

Supplementary Examination 2017/2018
COURSE NAME: Electromagnetic Theory
COURSE CODE: PHY332/P331
TIME ALLOWED: 3 hours

ANSWER ANY FOUR QUESTIONS. ALL QUESTIONS CARRY EQUAL (**25**) MARKS

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The exam paper has nine (9) printed pages, including an appendix.

Question 1

(a) Determine the electrostatic field intensity $\vec{E}$ at the point $(1, 1, 0)$ for the following scalar electric potentials

See comment in main examination.

(i) $\Phi = E_0 e^{-x} \sin\left(\frac{\pi y}{4}\right)$. [3 marks]

(ii) $\Phi = E_0 R \cos(\theta)$. [3 marks]

(b) A conical surface, with height h has a radius, also equal to the height. The uniform surface of the cone carries a uniform charge σ.

(i) Find the potential, $\Phi(\mathrm{a})$, at the vertex point a. [5 marks]

(ii) Find the potential, $\Phi(\mathrm{b})$, at the centre top b. [5 marks]

(i) Show that $\Phi(\mathrm{b}) - \Phi(\mathrm{a}) = \frac{\sigma h}{2\epsilon_0}\left[1 - ln(1+\sqrt{2})\right]$ [3 marks]

(c) Two infinite grounded metal plates lie parallel to the xz-plane, one at $y = 0$, the other at $y = a$ as ahown in figure 1. The left end, at $x = 0$, is closed off with an infinite strip insulated from the two plates and maintained at a specific potential, $V_0(y)$.

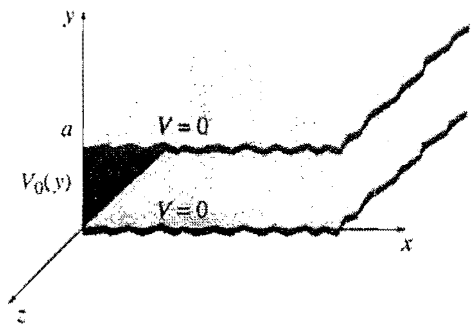

Figure 1:

(i) Use separation of method to find a general solution for the x- and y- components of the potential inside the two plates.
[3 marks]

(ii) After applying the boundary conditions, the general solution can be written as

$$V(x, y) = \sum_{n=1}^{\infty} C_n e^{-\left(\frac{n\pi x}{a}\right)} \sin\left(\frac{n\pi y}{a}\right). \tag{1}$$

Employ Fourier's trick to find the coefficients C_n for $V_0(y) = V_0$. Write the expression for $V(x, y)$ for this potential.

[3 marks]

Question 2

(a) A conducting sphere of radius a is held at a constant potential V_0 with respect to infinity. At a distance a from its surface ($2a$ from its center) the potential is $\frac{V_0}{2}$. Calculate the potential at a distance $a/2$ from its surface ($1.5a$ from its center).[**6 marks**]

(b) Two point charges are placed in free space at a distance d from each other in free space. One charge is positive and equal to Q, the second is negative and equal to $-Q$ as shown in Figure 2 A. Point P is the center point between the two charges. Now a conductor is brought into the vicinity of the two charges so that the configuration is as in Figure 2 B.

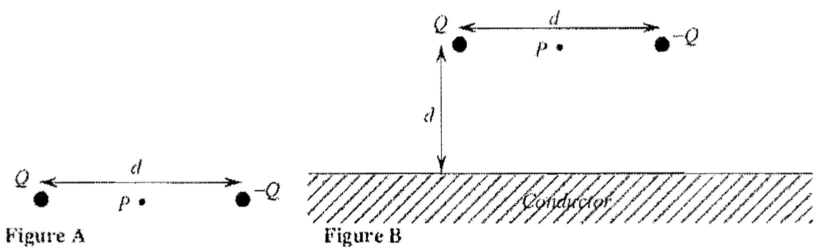

Figure 2:

(i) Find the change in the electric field intensity at point P caused by the conductor.[**9 marks**]

(ii) Calculate the change in the potential at point P caused by the conductor.[**3 marks**]

(c) A solid conductor of radius b and length L has a known conductivity σ. Two holes are now drilled into the conductor, parallel to the axis of the conductor, each hole being of radius a ($a < b/2$) (see Figure 3). Calculate the change in resistance of the device due to the drilling of the holes. The resistance is calculated along the cylinder (i.e. as if the current flows along the cylinder).[7 **marks**]

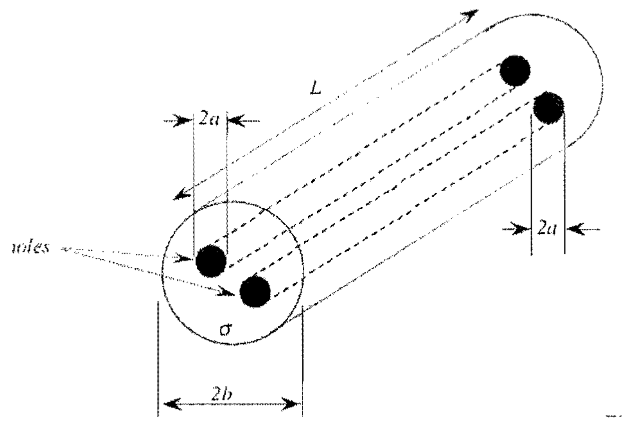

Figure 3:

Question 3

electric

(a) A sphere of radius R contains a charge whose density is $\rho(r,\theta) = kcos(\phi)$, where r, θ, and ϕ are the standard spherical coordinates. Determine the lecetric dipole moment. (*Hint:Consider the symmentry of the charge distribution*)[**10 marks**]

(b) An infinitely long cylindrical shell, ~~of negligible~~ thickness and radius b, sarrounds an infinitely long solid cylindrical rod of radius a. Their axis are both along the z-axis. A view ~~down the~~ length of this axis is shown in figure 4. The volume current density in the rod is $\vec{J} = \frac{3\alpha}{2\pi a^3} s\hat{z}$, where α is a constant with dimensions of current. The current along the cylindrical shell, flows in the $-\hat{z}$ direction, is uniformly distributed across the surface and is such that the total current flowing down the shell is exaclty opposite to that flowing through the cylinder.

surrounds

exactly

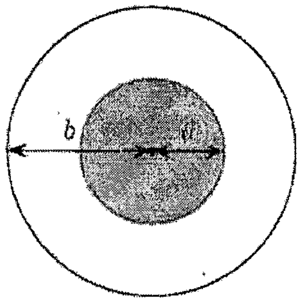

Figure 4:

(i) Show that the surface charge density in the cylindrical shell is $\vec{K} = -\frac{\alpha}{2\pi b}\hat{z}$.[**8 marks**]

(ii) Determine the magnetic field at all locations.[**7 marks**]

Question 4

(a) An infinitely long cylindrical shell extending between $1m \leq r \leq 3m$ contains a uniform charge density ρ_0. By applying Guass's Law , find $\overrightarrow{D}$ in the shell region.[**5 marks**]

(b) A cylindrical conductor whose axis is coincident with the x-axis has a radius a and carries a current characterised by a current density $\overrightarrow{J} = \hat{x}\dfrac{J_0 + 1}{r}$, where J_0 is a constant and r is the radial distance from the cylinder's axis. Obtain an expression for the magnetic field $\overrightarrow{H}$ for $r > a$. [**10 marks**]

(c) Find θ_1 for figure 5, if $\overrightarrow{E_2} = 5\hat{x} - \hat{y} + 2\hat{z}\ V/m$, $\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 = 2\varepsilon_0$ and for a boundary with a surface charge density of $\rho_s = 2.2 \times 10^{-11}\ C/m^2$.[**10 marks**]

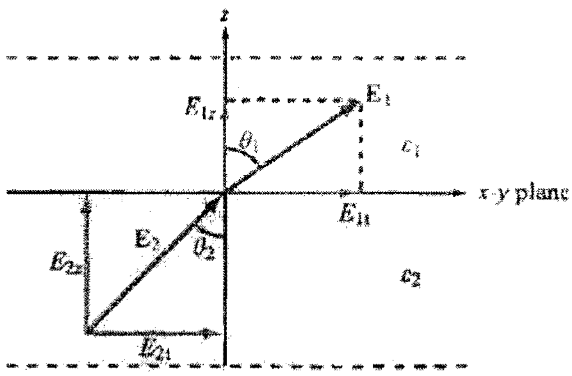

Figure 5:

Question 5

(a) Suppose the circuit in figure 6 has been connected for a long time when suddenly, at $t = 0$, the switch S is thrown, bypassing the battery.

(i) What is the current at any subsequent time t? **[5 marks]**

(ii) What is the total energy delivered to the resistor? **[6 marks]**

(iii) Show this energy is equal to the energy originally stored in the inductor. **[4 marks]**

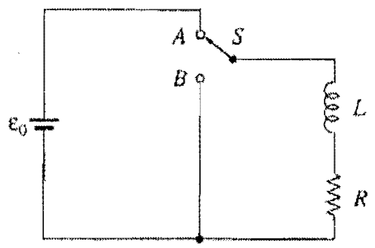

Figure 6:

(b) Sea water at frequency $\nu = 4 \times 10^8\ Hz$ has a permittivity $\epsilon = 81\epsilon_0$, permeability $\mu = \mu_0$ and resistivity $\rho = 0.23\ \Omega \cdot m$. Consider a parallel plate capacitor immersed in sea water and driven by a voltage $V_0 \cos(2\pi\nu t)$, What is the ratio of conduction current to displacement current? **[10 marks]**

Appendix

Some useful information:

1(a) dipole potential

$$\Phi_{dipole} = \frac{1}{4\pi\epsilon_0}\frac{\hat{\mathrm{p}}\cdot\hat{\mathrm{r}}}{r^2}$$

1(b) dipole

$$\hat{\mathrm{p}} = i\int_v r\rho(r)d\tau$$

2 for Legendre polynomials

$$\begin{aligned}\int_0^\pi P_\ell(\cos\theta)P_{\ell'}(\cos\theta)\sin\theta d\theta &= 0 && \text{if } \ell' \neq \ell \\ &= \frac{2}{2\ell+1} && \text{if } \ell' = \ell\end{aligned}$$

3 Maxwell's general equations;

$$\begin{aligned}\nabla\times\overrightarrow{E} &= -\frac{\partial\overrightarrow{B}}{\partial t} \\ \nabla\times\overrightarrow{H} &= \overrightarrow{J} + \frac{\partial\overrightarrow{D}}{\partial t} \\ \nabla\cdot\overrightarrow{D} &= \rho \\ \nabla\cdot\overrightarrow{B} &= 0\end{aligned}$$

where $\overrightarrow{D} = \epsilon\overrightarrow{E}$, and $\overrightarrow{H} = \frac{1}{\mu}\overrightarrow{B}$

4 Maxwell's equations in a nonconducting medium;

$$\begin{aligned}\nabla\times\overrightarrow{E} &= -\mu\frac{\partial\overrightarrow{H}}{\partial t} \\ \nabla\times\overrightarrow{H} &= \epsilon\frac{\partial\overrightarrow{E}}{\partial t} \\ \nabla\cdot\overrightarrow{D} &= 0 \\ \nabla\cdot\overrightarrow{B} &= 0\end{aligned}$$

5

$$\overrightarrow{A}\times(\overrightarrow{B}\times\overrightarrow{C} = \overrightarrow{B}(\overrightarrow{A}\cdot\overrightarrow{C}) - \overrightarrow{C}(\overrightarrow{A}\cdot\overrightarrow{B})$$

6 Permeability of free space $\mu_0 = 1.25663706 \times 10^{-6}(H/m)$.

7 Permittivity of free space $\epsilon_0 = 8.854 \times 10^{-12}(F/m)$.

8 Gauss's theorem:

$$\int_v \nabla \cdot \overrightarrow{A} dv = \oint_c \overrightarrow{A} \cdot d\overrightarrow{s}$$

9 Stoke's theorem:

$$\int_s \nabla \times \overrightarrow{A} \cdot d\overrightarrow{s} = \oint_c \overrightarrow{A} \cdot d\overrightarrow{\ell}$$

10

$$\int \overrightarrow{B} \cdot d\overrightarrow{l} = \mu_0 I_{enclosed}$$