UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

RE-SIT/SUPPLEMENTARY EXAMINATION: 2018/2019

TITLE OF PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: PHY221/P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE WHEN NECESSARY.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Useful Mathematical Relations

Gradient Theorem

$$\int_{\vec{a}}^{\vec{b}} (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$$

Divergence Theorem

$$\int \nabla \cdot \vec{A} d\tau = \oint \vec{A} \cdot d\vec{a}$$

Curl Theorem

$$\int (\nabla \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

Line and Volume Elements

Cartesian: $d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$, $d\tau = dxdydz$

Cylindrical: $d\vec{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$, $d\tau = sdsd\phi dz$

Spherical: $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$, $d\tau = r^2 \sin\theta dr d\theta d\phi$

Gradient and Divergence in Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Dirac Delta Function

$$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$$

Question 1: Electrostatics...

A solid conducting sphere of radius a is surrounded by a thin conducting spherical shell of inner radius b, with $b > a$. The inner sphere carries a charge Q and the outer sphere carries a charge $-Q$.

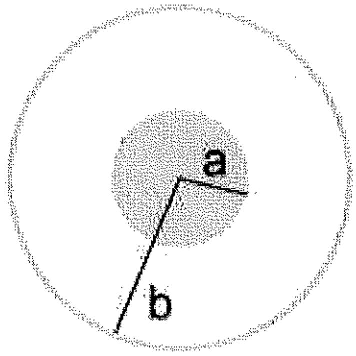

Figure 1: A sketch of a spherical capacitor.

(a) Using Gauss' Law, find an expression for the electric field $\mathbf{E}$ at points $a < r < b$. Show the Gaussian surface you use. (8)

(b) Using your expression for $\mathbf{E}$, find the electric potential difference between the inner and outer conductor. (6)

(c) Using your results for the electric potential difference, determine the capacitance of this capacitor, in terms of the quantities given. (3)

(d) Use your expression for $\mathbf{E}$ to calculate the total electrostatic energy U by integrating the energy density $dU = \frac{1}{2}\epsilon_0 E^2$ over the volume. Recall that the volume element in spherical coordinates is $d\tau = r^2 \sin\theta dr d\theta d\phi$. (4)

(e) Show that the energy you get agrees with either $\frac{1}{2}CV^2$ or $\frac{1}{2}\frac{Q^2}{C}$. (4)

Question 2: Electrostatic II ....

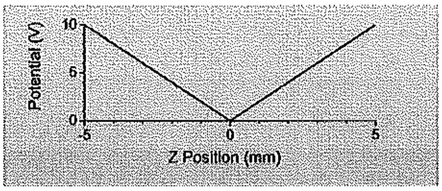

Figure 2: The potential V depends on the coordinate z and not on x and y.

(a) Consider the potential V shown on Fig. 2.

i. Use the potential to determine the field everywhere. Hint: Determine the functional of V for $z < 0$ separately from its form for $z > 0$. (10)

ii. Are there charges anywhere? If so what sign? (5)

(b) Briefly describe five electric properties of ideal conductors. (10)

Question 3: Magneto-statics ....

A coaxial cable consists of an inner conductor of radius a, surrounded by a conducting concentric cylindrical tube of inner radius b and outer radius c. The conductors carry equal and opposite currents I_0 distributed uniformly across their cross-sections as shown in Fig. 3.

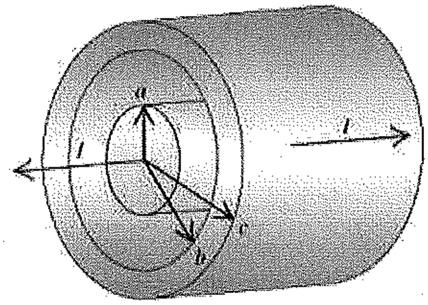

Figure 3: Sketch of the cross-section of a coaxial cable.

(a) Determine the magnetic field for points in each of the following ranges (Show the Amperian loop used in each case).

   i. $r < a$, (6)

   ii. $a < r < b$, (4)

   iii. $b < r < c$, (6)

   iv. $r > c$ (4)

(b) Sketch the magnitude of the magnetic field as a function of the distance r from the axis of the cable. (5)

Question 4: Magnetostatics II ....

(a) In the following steps we calculate the self inductance L of a solenoid. Assume the solenoid has length l, radius R and n turns per meter. Assume $L >> R$.

i. Assuming a current I flows in the solenoid, determine the field $\mathbf{B}$. (6)

ii. Calculate the flux due to the field $\mathbf{B}$. (4)

iii. Calculate L by dividing the flux with the current. (2)

(b) Consider a rectangular loop inside a uniform magnetic field as shown in Fig. 4.

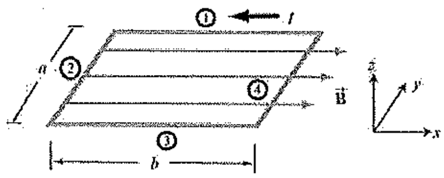

Figure 4: A current carrying loop inside a magnetic field.

i. What is the net force on the loop? (4)

ii. What is the net torque on the loop? (4)

iii. Describe the motion of the loop. (5)

Question 5: Circuits and Electrodynamics ....

(a) A capacitor C, which has been charged up to potential V_0, is connected to a resistor at time $t = 0$.

i. Determine the charge on the capacitor as a function of time, $Q(t)$. Include a circuit diagram as a starting point of your solution. (8)

ii. Determine the current through the resistor as a function of time, $I(t)$. (4)

iii. Calculate the energy stored in the capacitor when fully charged. (2)

iv. Integrate $P = VI$ and confirm that the heat delivered to the resistor is equal to the energy lost by the capacitor. (6)

(b) Consider a battery that is part of a circuit. What moves the charges in all other parts of the circuit if driving force (emf) is in the battery? (i.e describe the mechanism responsible for the flow of charge in the circuit.) (5)