UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

RESIT EXAMINATION 2018/2019

TITLE OF PAPER:	WAVE OPTICS AND MODERN PHYSICS
COURSE CODE:	PHY232
TIME ALLOWED:	THREE HOURS

USEFUL INTSRUCTIONS:

1. There are five questions in this paper, and each question carries a total of 25 marks. Answer any four questions in your preferred order.
2. Additional materials included in this paper are a list of useful constants and the periodic table.

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THIS PAPER CONSISTS OF 10 PAGES WITH COVER PAGE AND ADDITIONAL BACK PAGE INCLUDED

Question One [25 marks]

(a) State *one* reason why the wave theory on the nature of light was initially rejected. (1)

(b) Suppose that a time-varying wave function takes the form $y(t) = A\sin(2\pi t/T)$, where T is the period. Show that $y(t+T) = y(t)$. (2)

(c) (i) The equation of the simple harmonic oscillator is given by $-kx = m\frac{d^2x}{dt^2}$. Show that its solution is given by $x(t) = A\cos(\omega t) + B\sin(\omega t) = C\cos(\omega t + \phi)$, where $C = \sqrt{A^2 + B^2} > 0$ and $\phi = \tan^{-1}\left(-\frac{B}{A}\right)$. (6)

(ii) Giving a symbolic expression for each quantity, define the following quantities in relation to the simple harmonic oscillator:

1. Frequency, f. (1)
2. Angular frequency, ω. (1)
3. Period, T. (1)

(d) If one drops a small stone in a calm pond of water, waves spread outwards in concentric circles from the point of impact. Every point on the water surface begins to oscillate, and all points in each of the concentric circles oscillate in phase. Using this information or otherwise:

(i) Define the term *wavefront*. (1)

(ii) State *Huygens principle*. (2)

(e) Determine the effect on the interference fringes in a Young's double slit experiment if:

(i) The screen is moved away from the plane of the slits. (2)

(ii) The source S is replaced by another source S' of a shorter wavelength. (2)

(iii) The separation d between the two slits is increased. (2)

(iv) The source slit S is moved closer to the two slits (S_1 and S_2) plane. (2)

(v) The width of the source slit S is increased. (2)

Question Two [25 marks]

(a) Fig. 2.1 is a schematic of the Michelson interferometer. Using all of the information depicted in the figure, or otherwise:

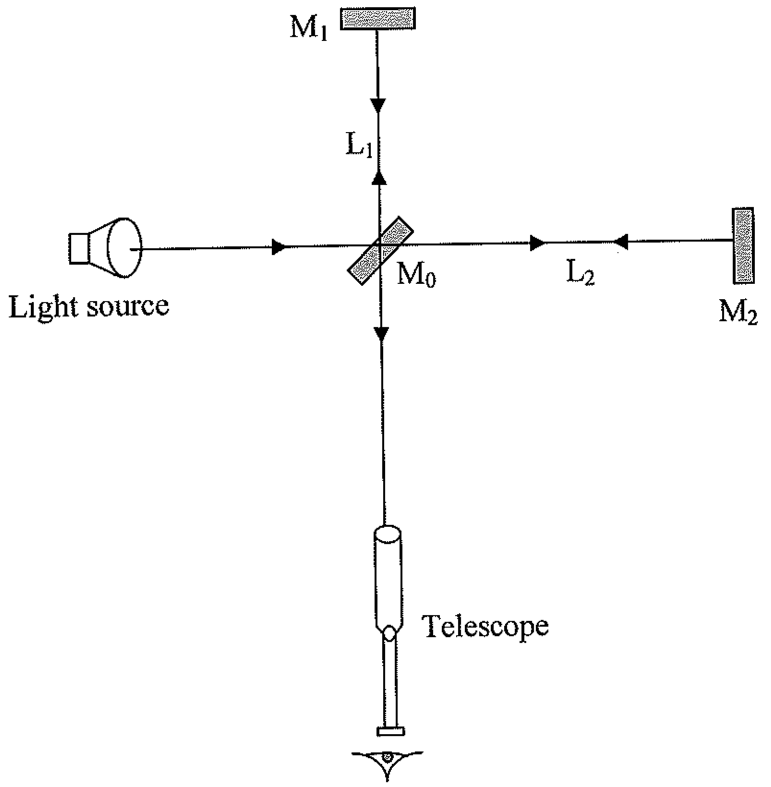

Fig. 2.1

(i) Describe the principle of operation of this device. (5)

(ii) State one use of this device, and describe how this is practically achieved. (5)

(b) Fig. 2.2 is a schematic of the setup of Young's double slit experiment. Suppose $d = 0.10$ mm and $D = 1.00$ m, and the incident light is monochromatic with wavelength λ=500 nm.

(i) What is the phase difference between the two waves arriving at a point P on the screen when $\theta = 0.8°$? (2.5)

(ii) What is the phase difference between the two waves arriving at a point P on the screen when $x = 4.00$ mm ? (2.5)

(iii) If the phase difference is $\phi = 1/3$ rad, what is the value of θ? (2.5)

(iv) If the path difference is $\delta = \lambda/4$, what is the value of θ? (2.5)

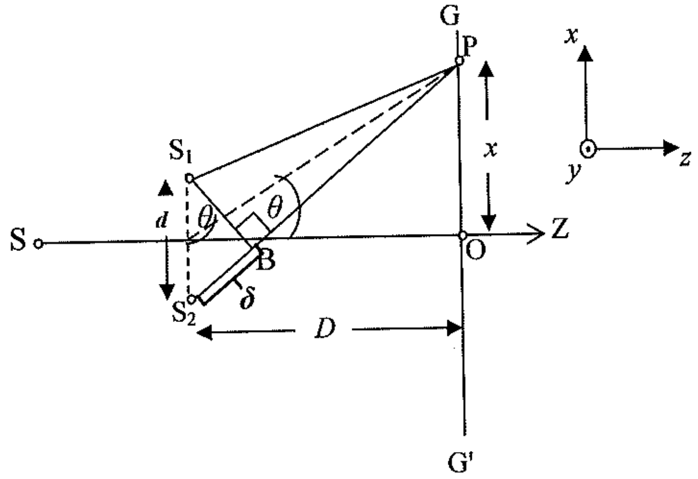

Fig. 2.2

(c) Fig. 2.3 shows light of wavelength $\lambda = 500\,\text{nm}$ incident on an interface such that $n_1 = 1.38$ and $n_2 = 1.50$. Calculate the wavelength of the light in the thin film of refractive index n_1 and further deduce the film thicknesses t that corresponds to maximum and minimum intensities in the fringe or interference pattern formed. (5)

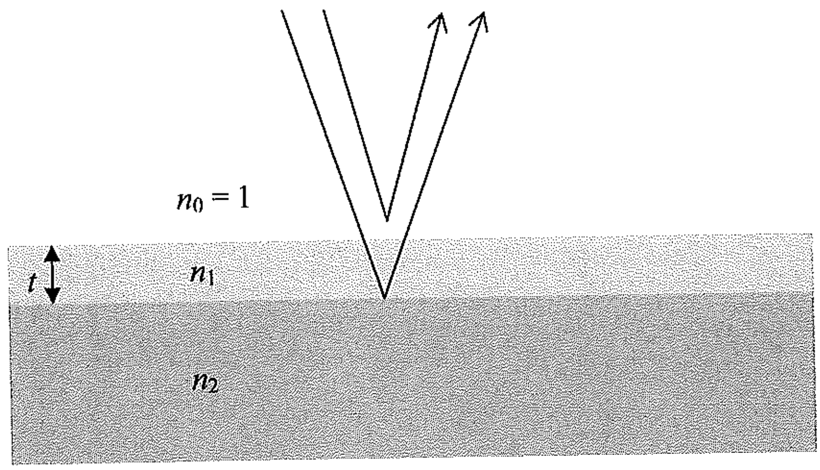

Fig. 2.3

Question Three [25 marks]

(a) With reference to the polarization of light waves, define the following words and/or phrases:

(i) Linearly polarized wave. (2)

(ii) Plane polarized wave. (2)

(iii) Polarizer. (2)

(iv) Fig. 3.1 shows an electromagnetic wave propagating at velocity $\vec{c}$ in the x-direction.

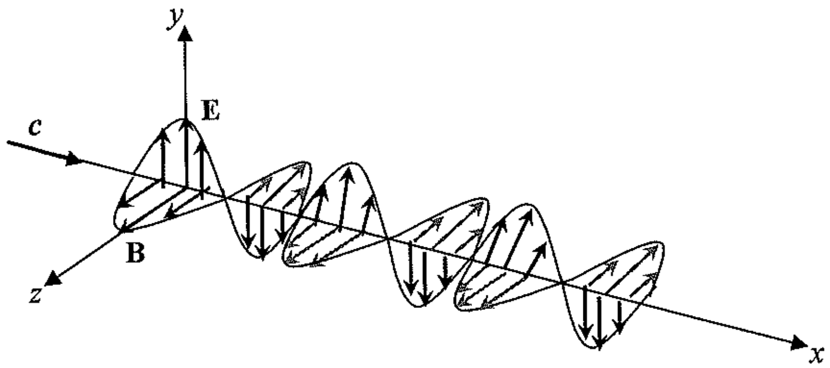

Fig. 3.1

1. If this was a light wave, in which direction would it be polarized? (1)
2. Based on your response above, write down the general equation of this wave. (1)
3. Suppose two linear polarizers are used such that the allowed polarization direction of the second one is at an angle θ to the first. Given that the $\vec{E}$ field of the first one is $\vec{E}(z,t) = \hat{x}E_0 \cos(kz - \omega t)$ and the $\vec{E}$ field in the preferred frame of the second one is given by $\vec{E}'(z,t) = E_0[\hat{x}'\cos\theta\cos(kz - \omega t) - \hat{y}'\sin\theta\cos(kz - \omega t)]$, state *Malus' law* in words and in equation form. (3)

(v) Fig. 3.2 shows the polarization of light by reflection when incident at an interface between two media of refractive indices n_1 and n_2. Given that θ_p is the angle of incidence at which the reflected beam is completely polarized, show that the refractive indices of the two media are related by $\frac{n_2}{n_1} = \frac{\sin\theta_p}{\cos\theta_p} = \tan\theta_p$. (5)

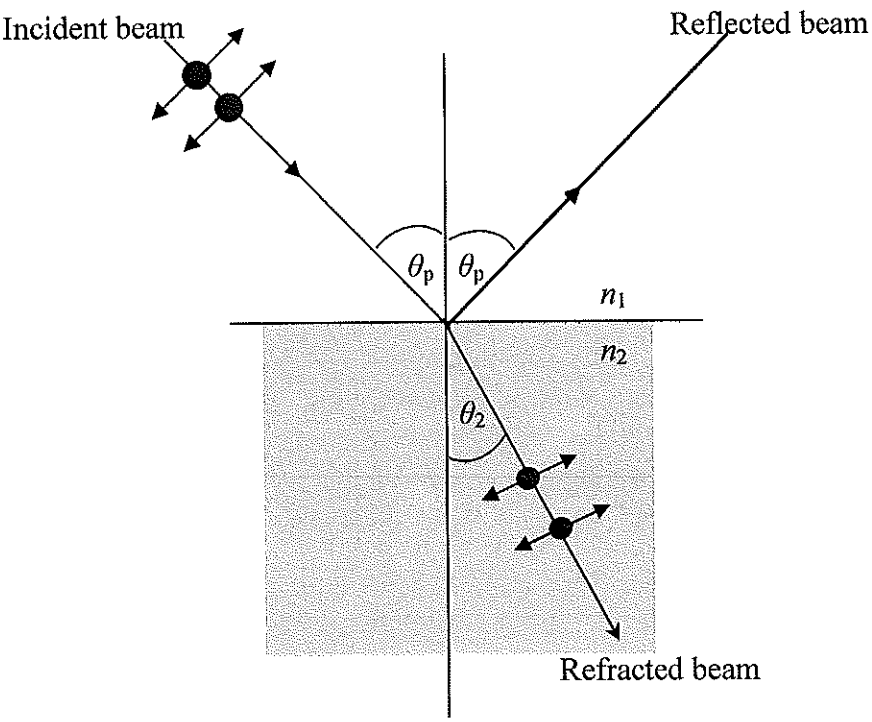

Fig. 3.2

(b) The wavelength of maximum intensity of the sun's radiation is found to be near 500 nm. Assume that the sun is a blackbody and calculate:

(i) The sun's surface temperature. (2)

(ii) The power per unit area $R(T)$ emitted from the sun's surface. (2)

(iii) The energy received by the Earth each day from the sun's radiation. (5)

{<u>Hint</u>: Sun's radius $r_S = 6.96\times10^5$ km. The fraction F of the sun's radiation received by Earth is

$F = \dfrac{\pi r_E^2}{4\pi R_{Es}^2}$, where $r_E = 6.37\times10^6$ m is the radius of Earth, and $R_{Es} = 1.49\times10^{11}$ m is the mean

Earth-sun distance.}

Question Four [25 marks]

(a) Planck's radiation law formulates the spectral distribution of blackbody radiation according to the relationship $I(\lambda,T)=\frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$, and Wien's displacement law explains the spectral distribution of blackbody radiation $\lambda_{max}T = 2.898\times10^{-3}\ \mathrm{m\cdot K}$. Show that Wien's displacement law follows from Planck's radiation law. (8)

(b) Raleigh-Jeans law models the spectral distribution of blackbody radiation by the relationship $I(\lambda,T)=\frac{2\pi ckT}{\lambda^4}$. Show that the Planck's radiation law agrees with the Raleigh-Jeans formula for long wavelengths. (5)

(c) (i) Determine the frequency of light needed to produce electrons of kinetic energy 3.00 eV from the illumination of lithium given that the work function of lithium is $\phi = 2.93\ \mathrm{eV}$. (3)

(ii) Find the wavelength of this light source and discuss where about in the electromagnetic spectrum it is located. (3)

(iii) A wavelength of 350 nm is used for light intensity of $I = 10^{-8}\ \mathrm{W\cdot m^{-2}}$. Determine the number of photons *per square meter per second* ($\mathrm{m^{-2}\cdot s^{-1}}$) in the light beam. (6)

Question Five [25 marks]

(a) An x-ray of wavelength $\lambda = 0.050\,\text{nm}$ scatters from a gold target.

(i) Can the x-ray be Compton-scattered from an electron bound by as much as 62 keV? (3)

(ii) What is the largest wavelength of a scattered photon that can be observed? (2)

(iii) What is the kinetic energy of the most energetic recoil electron and at what angle does it occur? (5)

(b) An atom in an excited state normally remains in that state for a very short time $\sim 10^{-8}$ s prior to emitting a photon and returning to the ground state. The lifetime of the excited state can be regarded as the uncertainty in time Δt.

(i) Calculate the characteristic *energy width* of such a state. (2)

(ii) Calculate the uncertainty ratio of the frequency $\Delta f / f$ if the wavelength of the emitted photon is $\lambda = 300\,\text{nm}$. (5)

(c) Fig. 5.1 is the schematic of a three-level laser. Describe the principle of operation of this laser system, further discussing on one major setback of such a system. (8)

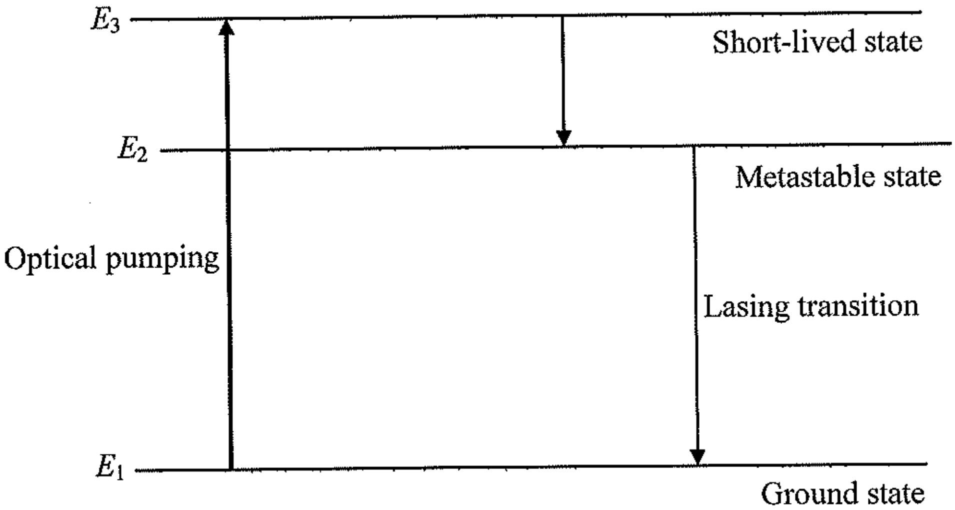

Fig. 5.1

Fundamental Constants

Quantity	Symbol	Value
Elementary charge	e	1.6022×10^{-19} C
Speed of light in vacuum	c	2.9979×10^{8} m/s
Permeability of vacuum	μ_0	$4\pi\times10^{-7}\ \mathrm{N\cdot A^{-2}}$
Permittivity of vacuum	ε_0	$8.8542\times10^{-12}\ \mathrm{F\cdot m^{-1}}$
Planck constant	h	$6.6261\times10^{-34}\ \mathrm{J\cdot s}$
		$4.1357\times10^{-15}\ \mathrm{eV\cdot s}$
Avogadro constant	N_A	$6.0221\times10^{23}\ \mathrm{mol^{-1}}$
Boltzmann constant	k	$1.3807\times10^{-23}\ \mathrm{J\cdot K^{-1}}$
Stefan-Boltzmann constant	σ	$5.6704\times10^{-8}\ \mathrm{W\cdot m^{-2}\cdot K^{-4}}$
Atomic mass unit	u	1.66053886×10^{-27} kg
		$931.494061\ \mathrm{MeV/c^2}$

Particle masses

Particle	**Mass in units of**		
	kg	**MeV/c²**	**u**
Electron	9.1094×10^{-31}	0.5110	5.4858×10^{-4}
Proton	1.6726×10^{-27}	938.27	1.00728
Neutron	1.6749×10^{-27}	939.57	1.00866
Deuteron	3.3436×10^{-27}	1875.61	2.01355
α particle	6.6447×10^{-27}	3727.38	4.00151

Conversion factors

$1\ \mathrm{MeV/c} = 5.344\times10^{-22}\ \mathrm{kg\cdot m/s}$	$1\ \mathrm{Ci} = 3.7\times10^{10}$ Bq
$1\ \mathrm{u} = 1.66054\times10^{-27}$ kg	$1\ \mathrm{eV} = 1.6022\times10^{-19}$ J

Useful Combinations of Constants

$\hbar = h/2\pi = 1.0546\times10^{-34}\ \mathrm{J\cdot s} = 6.5821\times10^{-16}\ \mathrm{eV\cdot s}$

$hc = 1.9864\times10^{-25}\ \mathrm{J\cdot m} = 1239.8\ \mathrm{eV\cdot nm}$

$\hbar c = 3.1615\times10^{-26}\ \mathrm{J\cdot m} = 197.33\ \mathrm{eV\cdot nm}$

$\dfrac{1}{4\pi\varepsilon_o} = 8.9876\times10^{9}\ \mathrm{N\cdot m^2\cdot C^{-2}}$

Compton wavelength $\lambda_C = \dfrac{h}{m_e c} = 2.4263\times10^{-12}$ m

$\dfrac{e^2}{4\pi\varepsilon_o} = 2.3071\times10^{-28}\ \mathrm{J\cdot m} = 1.4400\times10^{-9}\ \mathrm{eV\cdot m}$

Rydberg constant $R_\infty = 1.09737\times10^{7}\ \mathrm{m^{-1}}$

<u>The periodic table of elements with their atomic numbers</u>

I	II											III	IV	V	VI	VII	VIII
^{1}H																	^{2}He
^{3}Li	^{4}Be											^{5}B	^{6}C	^{7}N	^{8}O	^{9}F	^{10}Ne
^{11}Na	^{12}Mg											^{13}Al	^{14}Si	^{15}P	^{16}S	^{17}Cl	^{18}Ar
^{19}K	^{20}Ca	^{21}Sc	^{22}Ti	^{23}V	^{24}Cr	^{25}Mn	^{26}Fe	^{27}Co	^{28}Ni	^{29}Cu	^{30}Zn	^{31}Ga	^{32}Ge	^{33}As	^{34}Se	^{35}Br	^{36}Kr
^{37}Rb	^{38}Sr	^{39}Y	^{40}Zr	^{41}Nb	^{42}Mo	^{43}Tc	^{44}Ru	^{45}Rh	^{46}Pb	^{47}Ag	^{48}Cd	^{49}In	^{50}Sn	^{51}Sb	^{52}Te	^{53}I	^{54}Xe
^{55}Cs	^{56}Ba		^{72}Hf	^{73}Ta	^{74}W	^{75}Re	^{76}Os	^{77}Ir	^{78}Pt	^{79}Au	^{80}Hg	^{81}Tl	^{82}Pb	^{83}Bi	^{84}Po	^{85}At	^{86}Rn
^{87}Fr	^{88}Ra		^{104}Rf	^{105}Db	^{106}Sg	^{107}Bh	^{108}Hs	^{109}Mt	^{110}Ds	^{111}Rg	^{112}Cn	^{113}Nh	^{114}Fl	^{115}Mc	^{116}Lv	^{117}Ts	^{118}Og