UNIVESITY OF ESWATINI
FACULTY OF SCIENCE AND ENGINEERING
DEPARTMENT OF PHYSICS

Main Examination 2018/2019
COURSE NAME: Quantum Mechanics I
COURSE CODE: PHY341/PHY342
TIME ALLOWED: 3 hours

ANSWER ANY FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL (20) MARKS

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Question 1

Consider a one-dimensional bound particle.

(a). Show that

$$\frac{d}{dt}\int_{-\infty}^{\infty} \psi^*(x.t)\psi(x,t)dx = 0$$

[10 marks]

(b). Show that, if the particle is in a stationary state at a given time, then it will always remain in a stationary state.

[10 marks]

Question 2

Explain what was learned about quantization of radiation from the following experiments,

(a). The photo-electric effect.

[5 marks]

(b). Franck-Hertz experiment.

[5 marks]

(c). Compton scattering.

[5 marks]

(d). The black body radiation spectrum.

[5 marks]

Question 3

A particle of mass m is confined to a one-dimensional region, $0 \leq x \leq a$, as shown in figure 1. At $t = 0$, its normalised wave function is

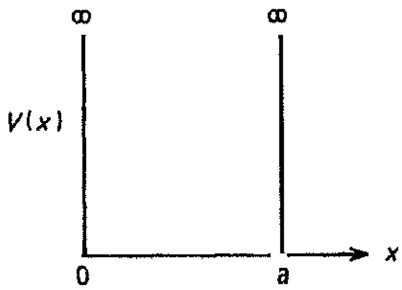


Figure 1:

$$\psi(x, t=0) = \sqrt{\frac{8}{5a}} \left[1 + \cos\left(\frac{\pi x}{a}\right)\right] \sin\left(\frac{\pi x}{a}\right).$$

(a). What is the wave function at a later time $t = t_0$?

[10 marks]

(b). What is the average energy of the system at $t = 0$ and at $t = t_0$?

[5 marks]

(c). What is the probability that the particle is found in the left half of the box, at $t = t_0$?

[5 marks]

Question 4

(a). Show that the expectation value of $\hat{p}$ obeys Newton's second law.

[8 marks]

(Hint: Prove Ehrenfest's thoerem that $\frac{d\langle\hat{p}\rangle}{dt} = \langle -\frac{dV}{dx}\rangle$*)*

(b). Show that the matrix $\mathbf{T} = \begin{pmatrix} 1 & 1-i \\ 1+i & 0 \end{pmatrix}$ is Hermitian.

[4 marks]

(c). An electron is confined in the ground state in a one-dimensional box of width $10^{-10}\ m$. Its energy is $38\ eV$. Calculate the energy of the electron in its first excited state.

[8 marks]

Question 5

For the operators $\mathbf{r}$ and $\mathbf{p}$, the canonical commutation relations are;

$$[r_i, p_j] = -[p_i, r_j] = i\hbar\delta_{ij}$$

$$[r_i, r_j] = [p_i, p_j] = 0.$$

Use the above relations to work out the following commutators;

(a). $[L_z, x]$

[2 marks]

(b). $[L_z, y]$

[2 marks]

(c). $[L_z, z]$

[2 marks]

(d). $[L_z, p_x]$

[2 marks]

(e). $[L_z, p_y]$

[2 marks]

(f). $[L_z, p_z]$

[2 marks]

(g). $[L_z, L_x]$

[2 marks]

(h). $[L_z, r^2]$, (where $r^2 = x^2 + y^2 + z^2$)

[3 marks]

(i). $[L_z, p^2]$, (where $p^2 = p_x^2 + p_y^2 + p_z^2$)

[3 marks]

Question 6

(a). A infinite cubical well ("or particle in a box") has a potential, defined in Cartesian 3D coordinates as

$$V(x,y,z) = \begin{cases} 0, & \text{if } x,y,z \text{ are all between 0 and a} \\ \infty, & \text{otherwise} \end{cases}$$

(i). What is the time-dependant Schrödinger equation for the particle inside the box

[5 marks]

(ii). Use the separation of variables ansatz ($\Psi(x,y,z) = X(x)Y(y)Z(z)$) to derive **three** separate ordinary differential equations for X, Y and Z in terms of K_x, K_y and K_z, where $E = \frac{\hbar^2}{2m}(K_x^2 + K_y^2 + K_z^2)$.

[5 marks]

(iii). Find solutions to the equations above. Apply the boundary conditions to show that

$$K_x a = n_x \pi$$
$$K_y a = n_y \pi$$
$$K_z a = n_z \pi$$

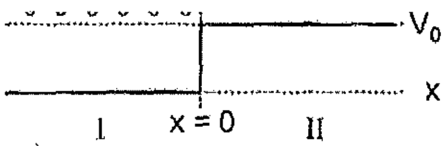
where $n_x, n_y, n_z = 1, 2, 3, ...$

[5 marks]

(b). Consider a step potential;

$$V(x) = \begin{cases} 0, & \text{if } x < 0 \\ V_0, & \text{if } x \geq 0 \end{cases}.$$

Consider the initial condition where a single particle of energy $E > V_0$ is incident from the left and no particle is incident from the right. Use



$k = \frac{\sqrt{2mE}}{\hbar}$ and $l = \frac{\sqrt{2m(E - V_0)}}{\hbar}$ to show that the time-independant Schrödinger equation for region I and II are

$$\frac{\partial^2 \Psi_I(x)}{\partial x^2} + k^2 \Psi_I(x) = 0$$
$$\frac{\partial^2 \Psi_{II}(x)}{\partial x^2} + l^2 \Psi_{II}(x) = 0.$$

[5 marks]

Appendix

Some useful information:

1. Planck's constant $h = 6.663 \times 10^{-34} J \cdot s$
2. Dirac's constant $\hbar = 1.05 \times 10^{-31} J \cdot s$
3. Permittivity of vaccum $\epsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$
4. Ground state hydrogen energy $-E_1 = 13.6057 eV$
5. Bohr energies $E_n = \frac{E_1}{n^2}$
6. Hydrogen ground state wavefunction $\Psi_0 = \frac{1}{\sqrt{\pi a^3}} e^{\frac{-r}{a}}$
7. $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

Y_0^0	$\left(\frac{1}{4\pi}\right)^{\frac{1}{2}}$
Y_1^0	$\left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \cos\theta$
$Y_1^{\pm}$	$\mp\left(\frac{3}{8\pi}\right)^{\frac{1}{2}} \sin\theta e^{\pm i\phi}$

Table 1: The first few spherical harmonics, $Y_l^m(\theta, \phi)$

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R_{10}	$2a^{\frac{-3}{2}} e^{\frac{-r}{a}}$
R_{20}	$\frac{1}{\sqrt{2}} a^{\frac{-3}{2}} \left(1 - \frac{r}{2a}\right) e^{\frac{-r}{2a}}$
R_{21}	$\frac{1}{\sqrt{24}} a^{\frac{-3}{2}} \left(\frac{r}{2}\right) e^{\frac{-r}{2a}}$

Table 2: The first hydrogen radial wavefunctions, $R_{nl}(r)$

8.