UNIVESITY OF ESWATINI
FACULTY OF SCIENCE AND ENGINEERING
DEPARTMENT OF PHYSICS

Supplementary Examination 2018/2019
COURSE NAME: Quantum Mechanics I
COURSE CODE: PHY341/PHY342
TIME ALLOWED: 3 hours

ANSWER ANY FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL (20) MARKS.
useful information and acronyms are given in the appendix, at the back.

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Question 1

A particle is in a infinite square well, whose potential is defined as

$$V(x) = \begin{cases} 0, & 0 \leq x \leq a \\ \infty, & otherwise \end{cases}$$

(a). What is the value of the wavefunction outside the well?

[2 marks]

(b). Show that the wavefunction inside the well is given by

$$\Psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right),$$

where $n = 1, 2, 3, ...$

[10 marks]

(c). Show that for the above wavefunction $\langle\Psi_1|\Psi_2\rangle = 0$ and $\langle\Psi_3|\Psi_3\rangle = 1$.

[6 marks]

(iv). What property is satisfied by the wavefunctions if $\langle\Psi_m|\Psi_n\rangle = \delta_{mn}$?

[2 marks]

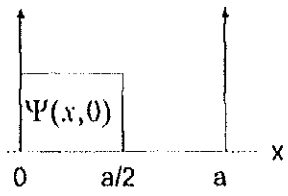

Figure 1:

Question 2

A particle of mass m in an infinite square well, of width a, starts out in the left of the well and is (at $t = 0$) equally likely to be found at any point in that region. This is shown in figure 1.

(a). Assuming the wave function is real, what is its initial value, $\Psi(x, 0)$?

[4 marks]

(b). The general solution to the TDSE at $t = 0$ is $\Psi(x,0) = \sum_{n=1}^{\infty} c_n \psi_n(x)$, where $\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$ form a complete set of orthonormal solutions for the TISE. Prove that $c_m = \langle \psi_m(x) | \Psi(x,0) \rangle$.

[6 marks]

(c). What is the expression for the probability P_n (in terms of c_n) that the measurement of $\Psi(x,0)$ collapses it into the n^{th} energy eigenstate, $\psi_n(x)$?

[4 marks]

(d). What is the probability that the measurement of the energy (at $t = 0$) would yield a ground state value, $E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$?

[6 marks]

Question 3

Explain what was learned about quantization of radiation from the following experiments,

(a). The photo-electric effect.

[5 marks]

(b). Franck-Hertz experiment.

[5 marks]

(c). Compton scattering.

[5 marks]

(d). The black body radiation spectrum.

[5 marks]

Question 4

A particle of mass m is confined to a one-dimensional region, $0 \leq x \leq a$, as shown in figure 2. At $t = 0$, its normalised wave function is

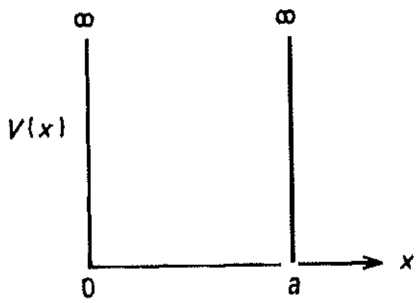

Figure 2:

$$\psi(x, t=0) = \sqrt{\frac{8}{5a}} \left[1 + \cos\left(\frac{\pi x}{a}\right)\right] \sin\left(\frac{\pi x}{a}\right).$$

(a). What is the wave function at a later time $t = t_0$?

[10 marks]

(b). What is the average energy of the system at $t = 0$ and at $t = t_0$?

[5 marks]

(c). What is the probability that the particle is found in the left half of the box, at $t = t_0$?

[5 marks]

Question 5

The ground state of the harmonic oscillator is $\psi_0(x) = \alpha e^{-\left[\frac{x^2}{2l^2}\right]}$, where $\alpha = \left(\frac{m\omega}{\pi\hbar}\right)^{(1/4)}$ and $\ell = \sqrt{\frac{\hbar}{m\omega}}$.

(a) Use symmetry and Ehrenfest's theroem to compute $\langle\hat{x}\rangle$ and $\langle\hat{p}\rangle$.

[6 marks]

(b). Compute $\langle\hat{x}^2\rangle$ and $\langle\hat{p}^2\rangle$.

[14 marks]

Question 6

Consider a one-dimensional bound particle.

(a). Show that

$$\frac{d}{dt}\int_{-\infty}^{\infty}\psi^*(x.t)\psi(x,t)dx = 0$$

[10 marks]

(b). Show that, if the particle is in a stationary state at a given time, then it will always remain in a stationary state.

[10 marks]

Appendix

Some useful information:

1. $\int_{-\infty}^{\infty} \xi^2 e^{-\xi^2} d\xi = \sqrt{\pi}/2$
2. $\int_{-\infty}^{\infty} e^{-\xi^2} d\xi = \sqrt{\pi}$

3 Planck's constant $h = 6.663 \times 10^{-34} J \cdot s$

4 Dirac's constant $\hbar = 1.05 \times 10^{-31} J \cdot s$

5 Permittivity of vaccum $\epsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$

6 Ground state hydrogen energy $-E_1 = 13.6057 eV$

7 Bohr energies $E_n = \dfrac{E_1}{n^2}$

8 Hydrogen ground state wavefunction $\Psi_0 = \dfrac{1}{\sqrt{\pi a^3}} e^{\frac{-r}{a}}$

Y_0^0	$\left(\frac{1}{4\pi}\right)^{\frac{1}{2}}$
Y_1^0	$\left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \cos\theta$
$Y_1^{\pm}$	$\mp \left(\frac{3}{8\pi}\right)^{\frac{1}{2}} \sin\theta e^{\pm i\phi}$

Table 1: The first few spherical harmonics, $Y_l^m(\theta, \phi)$

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R_{10}	$2a^{\frac{-3}{2}} e^{\frac{-r}{a}}$
R_{20}	$\frac{1}{\sqrt{2}} a^{\frac{-3}{2}} \left(1 - \frac{r}{2a}\right) e^{\frac{-r}{2a}}$
R_{21}	$\frac{1}{\sqrt{24}} a^{\frac{-3}{2}} \left(\frac{r}{2}\right) e^{\frac{-r}{2a}}$

Table 2: The first hydrogen radial wavefunctions, $R_{nl}(r)$

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11. Acronyms;

- TISE : time-independent Schrödinger equation
- TDSE : time-dependent Schrödinger equation