

UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2018/2019

TITLE OF THE PAPER: NUCLEAR PHYSICS

COURSE NUMBER: PHY441/P442

TIME ALLOWED: THREE HOURS

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**INSTRUCTIONS:**

- ANSWER ANY FOUR OUT THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 MARKS.
- MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION GIVEN IN THE ATTACHED APPENDIX WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

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**General Data:**

$$1 \text{ unified mass unit (u)} = 1.6605 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}/c^2$$

$$\text{Planck's constant } h = 6.63 \times 10^{-34} \text{ Js}$$

$$\text{Boltmann's constant } k = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

$$\text{Avogadro's number } 6.022 \times 10^{23} \text{ (g-mole)}^{-1}$$

$$\text{speed of light (vacuum) } c = 3.0 \times 10^8 \text{ m/s}$$

$$\text{electron mass} = 9.11 \times 10^{-31} \text{ kg} = 5.4858 \times 10^{-4} \text{ u} = 0.511 \text{ MeV}/c^2$$

$$\text{neutron mass} = 1.6749 \times 10^{-27} \text{ kg} = 1.008665 \text{ u} = 939.573 \text{ MeV}/c^2$$

$$\text{proton mass} = 1.6726 \times 10^{-27} \text{ kg} = 1.0072765 \text{ u} = 938.280 \text{ MeV}/c^2$$

$$1 \text{ year} = 3.156 \times 10^7 \text{ s}$$

$$\text{nuclear radius, } R \cong r_0 A^{1/3}, \text{ where } r_0 = 1.2 \text{ fm}$$

$$\text{fine structure constant } \alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = 1/137$$

$$\text{Planck constant } \hbar c = 1.973 \times 10^{-13} \text{ MeV} \cdot \text{m}$$

The table of nuclear properties is provided in the following page.

Nuclide	Z	A	Atomic mass (u)	$I^\pi$	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	$0^+$	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8s( $\beta^-$ )
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.000000	$0^+$	99.89%
N	7	15	15.000109	$1/2^-$	0.366%
N	7	18	18.014081	$1^-$	0.63 s
O	8	15	15.003065	$1/2^-$	122 s (e)
O	8	16	15.994915	$0^+$	99.76%
O	8	18	17.999160	$0^+$	0.204%
F	9	18	18.000937	$1^+$	110.0 min
Ne	10	20	19.992436	$0^+$	90.51%
Ne	10	22	21.991383	$0^+$	9.33%
Na	11	22	21.994434	$3^+$	2.60 yrs
Mg	12	22	21.000574	$0^+$	3.86 s
Al	13	27	26.981539	$5/2^+$	100.00 %
Si	14	22	29.973770	$0^+$	3.10%
Si	14	32	31.974148	$0^+$	105y
P	15	30	29.978307	$1^+$	2.50min
P	15	32	31.971725	$1^+$	14.3d
S	16	32	31.972071	$0^+$	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 d
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 d ( $\beta^-$ )
Sc	21	47	46.952409	$7/2^-$	3.35 d ( $\beta^-$ )
Fe	26	56	55.934439	$0^+$	91.8%
Fe	26	60	59.934078	$0^+$	1.5My
Co	27	60	59.933820	$5^+$	5.27y
Ni	28	60	59.930788	$0^+$	26.1%
Ni	28	64	63.927968	$0^+$	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 h ( $\beta^-$ )
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	$1^+$	12.7 h
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	$0^+$	48.6%
Ru	44	104	103.905424	$0^+$	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44h ( $\beta^-$ )
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 y ( $\beta^-$ )
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 d ( $\beta^-$ ) %
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%

### Question 1: General properties of nuclear decay

- (a) Consider an imaginary element called **Eswatinium** which as the isotopic notation given as  ${}_{121}^{247}\text{Es}$
- (i)  ${}_{121}^{247}\text{Es}$  decays by alpha decay to another element, **Kwalusenium**, Km: Write a balanced equation in isotope notation to describe this decay. [2 marks]
- (ii) Explain why the alpha decay is the most likely decay for this element. [2 marks]
- (b) **Kwalusenium** is unstable, decays by beta minus decay to an isotope of **Ngwanium**, Nm. Write down a balanced equation for this decay in isotope notation. [2 marks]
- (c) A sample of **Eswatinium** consists of  $3 \times 10^{11}$  atoms. It emits alpha particles and its activity is worked out to be  $1 \times 10^8$  Bq.
- (i) Calculate the decay constant,  $\lambda$ . [4 marks]
- (ii) Calculate the half life of **Eswatinium**. [3 marks]
- (d) **Eswatinium** also emits gamma rays as it decays to **Kwalusenium**. Explain how gamma rays are produced in the daughter nucleus. [2 marks]
- (e) Calculate the radius of **Eswatinium** ( $r_0 = 1.2$  fm). [3 marks]
- (f) For the nuclide  ${}^{16}\text{O}$  the neutron and proton separation energies are 15.7 and 12.2 MeV respectively. Estimate the radius of this nucleus assuming that the particles are removed from its surface and that the difference in separation energies is due to the Coulomb potential energy of the proton. [7 marks]

## Problem 2: Shell model

1. Find the configuration of the protons and neutrons in the incomplete shells and hence the ground state spin and parity assignments for the following nuclei

$${}^7_3\text{Li}; \quad {}^{23}_{12}\text{Na}; \quad {}^{33}_{16}\text{S}$$

under the assumption that the ordering of the lowest single particle nuclear energy levels is  $1s_{1/2}$ ;  $1p_{3/2}$ ;  $1p_{1/2}$ ;  $1d_{5/2}$ ;  $2s_{1/2}$ ;  $1d_{3/2}$ ;  $1f_{7/2}$ ;  $2p_{3/2}$

[9 marks]

2. In this model determine the spin and parity for the most likely first excited state for each of the three given nuclides:

$${}^7_3\text{Li}; \quad {}^{23}_{12}\text{Na}; \quad {}^{33}_{16}\text{S}$$

[9 marks]

3. The observed nuclear spin, parity and magnetic moment of  ${}^{43}_{20}\text{Ca}$  is:

$$I = 7/2^+; \mu = -1.32\mu_N$$

Determine the expected values for these moments according to the simple Shell model and comment on any significant differences.

[7 marks]

## Question 3: Beta decay

(a) Find out by actual calculations if  ${}_{27}^{60}\text{Co}$  can decay by

- (i)  $\beta^-$  emission,
- (ii)  $\beta^+$  emission,
- (iii) electron capture.

[9 marks]

(b) Sketch the energy level scheme for a  $\beta^-$  emission,  $\beta^+$  emission, and an electron capture for unstable nuclide with mass  $m(A, Z)$ .

[6 marks]

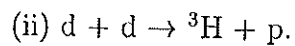
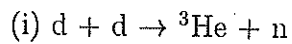
(c) Supply the missing particles in the following processes

- (i)  $\bar{\nu} + {}^3\text{He} \rightarrow$
- (ii)  $e^- + {}^8\text{B} \rightarrow$
- (iii)  ${}^{40}\text{K} \rightarrow \bar{\nu}$
- (iv)  $\nu + {}^{12}\text{C} \rightarrow$
- (v)  ${}^3_1\text{H} \rightarrow {}^3_2\text{He}$

[10 marks]

### Problem 4: Nuclear reactions

(a) Calculate the  $Q$  values for the fusion reactions



[8 marks]

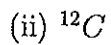
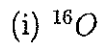
(b) Based on the estimate that the deuterons have to come within 100 fm of each other for fusion to proceed in the preceding reactions

(i) Calculate the energy that must be supplied to overcome the electrostatic repulsion.

(ii) Approximately what temperature is this equivalent to?

[7 marks]

(c) Consider a gas of atoms undergoing fusion. Calculate the temperature required to overcome the Coulomb barrier and the fusion energy release if the gas consists of



[10 marks]

### Problem 5: Excited states

- (a) For the following  $\gamma$  transitions, give all permitted multipoles and indicate which multipole might be most intense in the emitted radiation.

(i)  $\frac{9}{2}^- \rightarrow \frac{7}{2}^+$

(ii)  $\frac{1}{2}^- \rightarrow \frac{7}{2}^-$

(iii)  $1^- \rightarrow 2^+$

(iv)  $4^+ \rightarrow 2^+$

(v)  $\frac{11}{2}^- \rightarrow \frac{3}{2}^+$

[10 marks]

- (b) Explain why a transition from  $0^+$  to  $0^+$  will not allow any  $\gamma$  radiation.

[2 marks]

- (c) An even-Z, even-N nucleus has the following sequence of levels above its  $0^+$  ground state:

$$2^+(89\text{keV}), 4^+(288\text{keV}), 6^+(585\text{keV}), 0^+(1050\text{keV}), 2^+(1129\text{keV})$$

Draw an energy level diagram and show all reasonably probable  $\gamma$  transitions and their dominant multipole assignments.

[10 marks]

- (d) List three modes by which a photon can interact with matter.

[3 marks]