UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2018/2019

TITLE OF THE PAPER: NUCLEAR PHYSICS

COURSE NUMBER: PHY441

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 MARKS.
- MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION GIVEN IN THE ATTACHED APPENDIX WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

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General Data:

1 unified mass unit (u) =1.6605×10^{-27}kg = 931.5 MeV/c^2
Planck's constant $h = 6.63^{-34}$Js

Boltmann's constant $k = 1.38 \times 10^{-23} JK^{-1}$

Avogardo's number 6.022×10^{23} (g-mole)$^{-1}$

speed of light (vacuum) $c = 3.0 \times 10^8$ m/s

electron mass = 9.11×10^{-31}kg = 5.4858^{-4}u = 0.511 MeV/c^2

neutron mass = 1.6749×10^{-27}kg = 1.008665 u = 939.573 MeV/c^2

proton mass = 1.6726×10^{-31}kg =1.0072765 u =938.280 MeV/c^2

1 year = 3.156^7 s

nuclear radius, R $\cong r_0 A^{1/3}$, where $r_0 = 1.2$ fm

fine structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = 1/137$

Planck constant $\hbar c = 1.973 \times 10^{-13}$M eV$\cdot$ m

The table of nuclear properties is provided in the following page.

Nuclide	Z	A	Atomic mass (u)	I^π	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	0^+	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8s(β^-)
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.000000	0^+	99.89%
N	7	15	15.000109	$1/2^-$	0.366%
N	7	18	18.014081	1^-	0.63 s
O	8	15	15.003065	$1/2^-$	122 s (e)
O	8	16	15.994915	0^+	99.76%
O	8	18	17.999160	0^+	0.204%
F	9	18	18.000937	1^+	110.0 min
Ne	10	20	19.992436	0^+	90.51%
Ne	10	22	21.991383	0^+	9.33%
Na	11	22	21.994434	3^+	2.60 yrs
Mg	12	22	21.000574	0^+	3.86 s
Al	13	27	26.981539	$5/2^+$	100.00 %
Si	14	22	29.973770	0^+	3.10%
Si	14	32	31.974148	0^+	105y
P	15	30	29.978307	1^+	2.50min
P	15	32	31.971725	1^+	14.3d
S	16	32	31.972071	0^+	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 d
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 d (β^-)
Sc	21	47	46.952409	$7/2^-$	3.35 d (β^-)
Fe	26	56	55.934439	0^+	91.8%
Fe	26	60	59.934078	0^+	1.5My
Co	27	60	59.933820	5^+	5.27y
Ni	28	60	59.930788	0^+	26.1%
Ni	28	64	63.927968	0^+	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 h (β^-)
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	1^+	12.7 h
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	0^+	48.6%
Ru	44	104	103.905424	0^+	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44h (β^-)
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 y (β^-)
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 d (β^-) %
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%

Question 1: Shell Model

(a) The lowest energy levels in the Shell Model, in order of increasing energy are

$$1s_{1/2},\ 1p_{3/2},\ 1p_{1/2},\ 1d_{5/2},\ 2s_{1/2},\ 1d_{3/2},\ 1f_j,\ \ldots.$$

(i) What are the possible values of j for the $1f$ levels.

[2 marks]

(ii) What is the value of j for the lowest $1f$ level? Justify your answer.

[2 marks]

(iii) Determine the spin and parity of the ground state of both the ${}^{40}_{20}Ca$ and ${}^{41}_{20}Ca$ nuclides.

[8 marks]

(iv) In the Shell model, a 'spin-orbit' interaction splits all the energy levels except the 's-type' levels. Why do the s-type levels remain unsplit?

[1 marks]

(b) The low-lying energy levels of ${}^{13}C$ are the ground state $(\frac{1}{2}^-)$; 3.09MeV $(\frac{1}{2}^+)$; 3.68MeV $(\frac{3}{2}^-)$ and 3.85MeV $(\frac{5}{2}^+)$. Interpret these states according to the shell-model.

[12 marks]

Question 2: Semi-Empirical Mass Formula

(a) Consider the Semi-Empirical Mass Formula (SEMF) that describes the binding energy of a nuclide with A nucleons and Z protons as

$$B(Z,A) = a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4 (Z - \frac{A}{2})/A + \frac{(-1)^Z + (-1)^N}{2} a_5 A^{-1/2}.$$

Describe *briefly* the 'origin' of the various terms in the Semi-Empirical Mass Formula.Give physical reasons for their signs.

[10 marks]

(b) Using the SEMF find the expression for Z which minimizes the nuclei with a given value of A. (Hint: disregard for simplicity the last term in the SEMF)

[5 marks]

(c) Calculate the kinetic energy of the alpha particle emitted in the process $^{235}\text{U} \rightarrow \alpha + {}^{231}\text{Th}$. The binding energy of the alpha particle is 28.3 MeV and you may assume the following values (in MeV) for the five coefficients in the semi-empirical expression for the binding energy of heavier nuclei: $a_1 = 15.5$; $a_2 = 16.8$; $a_3 = 0.72$; $a_4 = 23$; $a_5 = 34$.

[10 marks]

Question 3: Nuclear decay

(a) A by-product of some fission reactors is ^{239}Pu which is an α-emitter with a half-life of 24,120 years. Consider 1 kg of ^{239}Pu at t=0.[Atomic mass of $^{239}Pu = 239.052163$u].

(i) What is the number of ^{239}Pu nuclei at t=0?

[3 marks]

(ii) What is the initial activity?

[3 marks]

(iii) For how long would you need to store Plutonium until it has decayed to a safe activity level of 0.1 Bq?

[5 marks]

(b) In stars slightly more massive than the Sun, hydrogen burning is carried out mainly by the CNO cycle, whose first step is $p +^{12}_{6} C \rightarrow^{13}_{7} N + \gamma$. Estimate the energy of the gamma (in MeV), assuming the two initial nuclei are essentially at rest. Justify any simplifying assumptions you make. [Atomic masses:$^{1}_{1}H = 1.007825u,^{12}_{6} C = 12.000000u,^{13}_{7} N = 13.005739u$].

[4 marks]

(c) Consider the nuclear fission reaction $n +^{235}_{92} U \rightarrow^{141}_{56} Ba +^{92}_{36} Kr + 3n$.

(i) Calculate the energy released (in MeV) in the reaction. [Atomic masses: $^{235}_{92}U = 235.043915u$, $^{141}_{56}Ba = 140.9139u$, $^{92}_{36}Kr = 91.8973u$. The neutron mass is 1.008665u].

[4 marks]

(ii) You wish to run a 1000MW power reactor using $^{235}_{92}U$ fission. How much $^{235}_{92}U$ is required for one day's operation?

[6 marks]

Question 4 - General nuclear properties

(a) Consider a simple phenomenological estimate for the dependence of the nuclear radius r_n on the number of nucleons A,

$$r_b = 1.2A^{1/3}\text{fm}$$

Explain briefly the physical reasons prompting this dependence.

[3 marks]

(b) Consider the two isotopes $^{15}_{8}$O and $^{15}_{7}$N. Compute their nuclear radius.

[2 marks]

(c) Explain which of the two isotopes $^{15}_{8}$O and $^{15}_{7}$N is supposed to be more stable.

[3 marks]

(d) Which is the most stable between a proton or an neutron?

[2 marks]

(e) Discuss the essential features of the strong force.

[3 marks]

(f) Write a short note on the following

(i) Geiger-Muller counter

(ii) Scintillation detector

[6 marks]

(g) Discuss three typical β-decay processes.

[6 marks]

Question 5: Beta decay

(a) The isotope $^{14}_{8}$O is a positron emitter, decaying to an excited state of $^{14}_{7}$N. The gamma rays from this latter have an energy of 2.313 MeV and the maximum energy of the positrons is 1.835 MeV. The mass of $^{14}_{7}$N is 14.003074 u and that of the electron is 0.000549 u.

(i) Write the equation for the decay of the oxygen isotope and sketch an energy level diagram for the process.

[3 marks]

(ii) Given that one unified mass unit (u) is equal to 931.502 MeV/c^2 find the mass of $^{14}_{8}$O.

[6 marks]

(b) The daughter nucleus of a given alpha emitter has several accessible excited states and so the kinetic energy of an emitted particle can have one of several possible values. For a particular heavy nucleus these values in MeV are:

$$5.545; 5.513; 5.486; 5.469; 5.443; 5.417; 5.389.$$

It is also observed that the daughter nuclei produced emit gamma rays with one of the following energies (in keV)

$$26; 32; 43; 56; 59; 99; 103; 125.$$

Use this information to sketch a decay scheme indicating the energy levels and marking the gamma ray transitions. You may assume that the most energetic alpha particle leaves the daughter nucleus in its ground state.

[10 marks]

(c) Given that the stable sodium isotope is $^{23}_{11}Na$, what type of radioactivity would you expect from

(i) ^{22}Na

(ii) ^{24}Na

[6 marks]