UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2018 /2019

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: PHY461/P461

TIME ALLOWED : THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

APPENDICES 1 AND 2 CONTAIN DEFINITE INTEGRALS AND PHYSICAL CONSTANTS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question one

(a) (i) Explain briefly what is meant by *statistical weight* of a system of particles. (3 marks)

(ii) What is the significance of *statistical weight* as regards the properties of the system? (2 marks)

(b) (i) A system has 4 distinguishable particles to be distributed among 6 non-degenerate energy levels each having energy 0, ϵ, 2ϵ, 3ϵ, 4ϵ and 5ϵ. Obtain the various macrostates of the system if its total energy is 5ϵ. (3 marks)

(ii) Find the distribution of these particles in the above example
1. if they are bosons.
2. if they are fermions.
Give reason/s for your answers. (2 + 2 marks)

(c) (i) What is meant by *degeneracy* of an energy level? (2 marks)

(ii) Find the degeneracy of an energy level having energy $E = kn^2$, k being a constant and $n^2 = n_x{}^2 + n_y{}^2 + n_z{}^2 = 14$.
n_x, n_y, n_z are quantum numbers corresponding to a quantum state. (4 marks)

(d) (i) Define *density of states* of a system of particles. (2 marks)

(ii) Calculate the *density of states* at an energy level of 2.06 eV for a system of fermions having volume 10^{-4} m^3. (4 marks)

(iii) Sketch a graph to show how *density of states* varies with energy. (1 mark)

Question Two

(a) The Maxwell-Boltzmann distribution function for a system of classical particles in thermal equilibrium is

$$n_s = g_s \exp(\alpha + \beta\varepsilon_s)$$

where the symbols have their usual meanings.

A classical non-degenerate system has 2000 particles arranged in 3 energy levels having energies 1 unit, 2 units and 3 units. The total energy of the system is 2600 units.

(i) Use the above distribution function to find the values of α and β for the system. (9 marks)

(ii) Use these values and the distribution function to find the occupation of the energy levels. (3 marks)

(iii) Verify your results numerically by finding the total number and total energy of the system. (2 marks)

(b) The differential form of Maxwell-Boltzmann distribution function in terms of speed is

$$n(v)dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT} v^2 dv$$

Use this equation to obtain

(i) the mean speed and (6 marks)

(ii) the most probable speed of the molecules in a classical gas. (5 marks)

Question Three

(a) Show that in a system of bosons, for the most probable configuration, the distribution function can be represented as

$$n_s = \frac{g_s}{e^{-(\alpha+\beta\varepsilon_s)} - 1}$$ where the symbols have their usual meanings.

Given: statistical weight of a system of bosons $W = \prod_s \frac{(g_s - 1 + n_s)!}{(g_s - 1)!\,n_s\,!}$

(12 marks)

(b) (i) State what each symbol represents in the Bose-Einstein condensation equation

$$\frac{N'}{N} = \left(\frac{T}{T_B}\right)^{3/2}$$ (2 marks)

(ii) Find the relationship between the number of particles N_0 in the ground state and the temperature T. (2 marks)

(iii) Draw a sketch to show how N_0 varies with temperature. (2 marks)

(c) In a Bose-Einstein condensation experiment 10^7 atoms of a metal having atomic weight 85.47 g/mol were cooled down to a temperature of 200 nK. The atoms were confined to a volume of 10^{-15} m^3.

(i) Calculate the condensation temperature T_B. (4 marks)

(ii) Calculate how many atoms were in the ground state at 200 nK (3 marks)

Question Four

(a) Derive the quantum mechanical expression for the spectral distribution of energy from a black body,

$$E(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

where the symbols have their usual meanings. (11 marks)

(b) (i) Use the above Planck's distribution function to show that the total energy radiated from the black body is proportional to the fourth power of the absolute temperature.
(See appendix for definite integrals) (8 marks)

(ii) Given that the proportionality constant obtained in the above expression for the total energy is equal to $\sigma(4/c)$, where σ is the Stefan-Boltzmann constant and c the velocity of light, calculate the value of σ. (6 marks)

Question Five

(a) (i) Define *Fermi energy* of a system of Fermions. (2 marks)

(ii) State two differences between *Fermi energy* and *Fermi level.* (2 marks)

(iii) What is the probability of occupation of an electron at the *Fermi level* of an intrinsic semiconductor. (1 mark)

(b) (i) Given the density of states for a system of Fermions,

$$g(\varepsilon)d\varepsilon = \frac{4\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$$

show that the Fermi energy is a function of the particle density of the system.(4 marks)

(ii) Calculate the Fermi energy of metal having electron density of 2×10^{28} m^{-3}. Express your answer in electron volt. (4 marks)

(c) With the help of an appropriate energy band diagram, show that the density of electrons in the conduction band of a semiconductor is given by the expression:

$$N_e = 2\left(\frac{2\pi mkT}{h^2}\right)^{3/2} \exp\left[\frac{\varepsilon_F - \varepsilon_g}{kT}\right]$$

where the symbols have their usual meanings. [Assume $(\epsilon - \epsilon_F) \gg kT$] (12 marks)

Appendix 1

Various definite integrals

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty e^{-ax}dx = \frac{1}{a}, (a > 0)$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} Js
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan- Boltzmann constant	σ	5.67×10^{-8} Wm^{-2} K^{-4}
Atmospheric pressure		1.01 10^5 Nm^{-2}
Mass of ${}_2^4$ He atom		6.65×10^{-27} kg
Mass of ${}_2^3$ He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 L mol^{-1}