UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2018/2019

TITLE OF THE PAPER: COMPUTATIONAL METHODS-II

COURSE NUMBER: PHY482/P482

TIME ALLOWED:
SECTION A: 1 HOUR
SECTION B: 2 HOURS

INSTRUCTIONS:
THE ARE TWO SECTIONS IN THIS PAPER:

- **SECTION A** IS A WRITTEN PART. ANSWER THIS SECTION ON THE ANSWER BOOK. IT CARRIES A TOTAL OF **40** MARKS.
- **SECTION B** IS A PRACTICAL PART FOR WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. IT CARRIES A TOTAL OF **60** MARKS.
- You may proceed to do Section B only after you have submitted your answer book for Section A

Answer **all** the questions from **section A** and **all** the questions from **section B**. Marks for different sections of each question are shown in the right hand margin.

THE PAPER HAS 6 PAGES, INCLUDING THIS PAGE.

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Section A

Question 1

(a) The following code fragment shows a potential infinite loop:

```
i =1
while (i >0) do
i =i+1
write(*,*), i**2
end do
```

Discuss two ways this do loop can perform the same operations but terminate after 100 runs, i.e., at i=100.

[2 marks]

(b) What is the name of the numerical differentiation scheme,

$$x_{n+1} = x_n + F(x_n)\Delta t,$$

approximating the solution to the ordinary differential equation $x'(t) = F(x(t))$. The error of this scheme $\epsilon \approx (\Delta t)^m$. What is the value of m?

[4 marks]

(c) Explain how you would use the above numerical differentiation scheme to solve the following equation

$$\ddot{y} = -\omega^2 y - f\dot{y}$$

where $\dot{y} = dy(t)/dt$.

[4 marks]

Question 2

a) Consider the following model equation,

$$\frac{\partial m(x)}{\partial t} = \frac{\partial^2 m(x)}{\partial x^2} + rm(x) - m^3(x)$$

in one dimension. Here r is constant. Show that this equation can be rewritten in the numerical difference scheme given as

$$m_i^{n+1} = m_i^n + \Delta t \cdot (m_{i+1}^n + m_{i-1}^n - 2m_i^n)/(\Delta x)^2 + \Delta t[rm_i^n - (m_i^n)^3]$$

where $m_i^n = m(i \cdot \Delta x, n \cdot \Delta t)$.

[6 marks]

b) Explain what would be the output of the following Fortran code:

```
C = 0.d0
L = 100
DO k =-L,L
DO j = -L,L
DO i =-L,L
N = i**2+j**2+k**2
if (N /= 0) then
C = C+(-1.0)**(i+j+k)/sqrt(1.0*N)
ELSE
ENDIF
END DO
END DO
END DO
```

[4 marks]

Question 3

a) Convert the following statements into valid F95 expressions

(i) $y = (2 - b)/x^3$

(ii) $g = \sum_{i=1}^{10} \sin(1.2 * i)$

(iii) $y = \begin{cases} 3x, & x \geq 0 \\ 2x - 1, & x < 0 \end{cases}$

(iv) $\pi = \tan^{-1}(-1.d0)$

[4 marks]

b) Given that for a single operation, single precision usually yields 6-7, decimal places for a 32-bit word, and double precision 12-16 decimal places. What would you guess is the result of the simple addition of two single precision numbers $4.500 + 3.0 \times 10^{-8}$:

(a) 4.50000003,

(b) 7.5×10^{-8}

(c) 4.5000000

(d) 4.8.

[2 marks]

c) Indicate the values that Fortran would produce in the following expressions:

i) $2 * *3 - 2$

ii) $4/3 - 1$

iii) $3./2$

iv) $1.d2 - 50$

[4 marks]

Question 4

a) Write a Fortran 95 function for

$$\text{sinc}(x) \equiv \frac{\sin x}{x}$$

Make sure that your function handles $x = 0$ correctly.

[5 marks]

b) Discuss how you would evaluate the following improper integral using the standard numerical methods such as the Simpson's Rule

$$I = \int_1^\infty \frac{dx}{1 + x^2}$$

[3 marks]

c) Name two condition under which the Monte Carlo integration method is most suitable compared to the standard numerical method such as the Trapezoidal and the Simpson rule.

[2 marks]

Section B

The answers to this question must include the computer code and output, in addition to any writing that might be needed.

Question 5

A point mass that can move along a straight line is attached to an end of an non-ideal elastic spring. (The other end of the spring is fixed.) A viscous friction force proportional to the velocity is acting on the mass. Therefore, the motion of the particle is described by the following differential equation:

$$\ddot{x} + \mu\dot{x} + x^3 = 0, \tag{1}$$

where μ is a positive dimensionless coefficient of nonlinear friction. Eq. (1) above is written in dimensionless units.)

(a) Write a code that solves this equation using the Euler method. Perform calculations with values $\mu = 0.08$, $\mu = 0.8$, and $\mu = 4$. Choose as initial conditions: $x(0) = 1$, $\dot{x}(0) = 0$.

[10 marks]

(b) Plot $x(t)$ versus t for the three values of μ in one graph.

[10 marks]

(c) Plot also $x(t)$ versus $\dot{x}(t)$ for the three values of μ in one graph. Which case corresponds to a damped, underdamped, and overdamped oscillatory motion?

[10 marks]

Question 6

Consider the following sequence $\{x_n\}$ defined as

$$x_i = (x_{i-1} + x_{i-2}) \bmod M \tag{2}$$

(a) Write a program that would generate $r_i = x_i/M$ in the range $i = 0..200$ for$M = 123$, $x_0 = 1$,and $x_1 = 8$. Plot r_i versus i for $i = 0..200$. Using your eye test can this be classified as good random number generator in this range. What is the period of this sequence.

[10 marks]

(b) Generate a plot of successive pair (r_{2i-1}, r_{2i}) for $i = 1..100$, the so-called scatter plot (Do not connect the points with lines.) Can this sequence be considered a random sequence at this range?

[10 marks]

(c) Perform a statistical test of uniformity by evaluating the mean of the distribution

$$\langle r_i^k \rangle = \frac{1}{N} \sum_{i=0}^{200} r_i^k \approx \frac{1}{k+1}$$

for $k = 1$ and $k = 2$. If the numbers are distributed *uniformly*, then $\langle r_i \rangle \approx 1/2$ and $\langle r_i^2 \rangle \approx 1/4$.

[10 marks]
