UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2019/2020

TITLE OF PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: PHY221/P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE WHEN NECESSARY.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

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Question 1: Electrostatics

(a) State the superposition principle for electric fields. (2)

(b) Starting from the field of a point charge, use the superposition principle to derive the general expression for a continuous charge distribution. (10)

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \rho(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}d\tau$$

(c) Consider a linear charge distribution of length $2a$ in which half the line has uniform charge density $-\lambda$ and the other half has uniform density $+\lambda$

i. Following similar steps to the derivation of the field of a continuous charge distribution, determine the electric field at a point z above the center of the charge distribution. (8)

ii. Determine the electric potential at the point where the field was determined. (5)

Note:

$$\int \frac{dx}{\sqrt{x^2+a^2}} = ln|x+\sqrt{x^2+a^2}| + C$$

Question 2: Electrostatics II

(a) Define an ideal conductor. (2)

(b) List the five basic electrostatic properties of an ideal conductor. (5)

(c) Consider three metallic concentric shells of radius $R_1 > R_2 > R_3$, respectively. The outermost shell carries charge q_1, the middle one q_2 and the smallest q_3.

i. Calculate the scalar potential V for all four regions, i.e. $r > R_1$ region, $R_2 < r < R_1$ region, $R_3 < r < R_2$ region and $r < R_3$ region. (12)

ii. Suppose that the outermost and innermost spheres are now connected by a very thin metallic wire. What is the charge (i.e. q_1', q_2' and q_3') on each shell now? (6)

Question 3: Magnetostatics

(a) A wire is formed into the shape of a square of edge length L. Assume the square is placed on the xy plane with its center at the origin.

i. Show, using the Biot-Sarvat law, that when the current in the loop is I, the magnetic field at a point a distance z above the center of the square, i.e the field point is on the z-axis, is (8)

$$B = \frac{\mu_0 I L^2}{2\pi(z^2 + L^2/4)\sqrt{z^2 + L^2/2}}$$

ii. Show that the field reduces to $2\sqrt{2}\mu_0 I/\pi L$ at the origin. (2)

(b) Consider two circular loops of radius R in which a current I flows. The loops have a common axis, let it be the z-axis, and they are placed such that they are separated by a distance d and the current flows in the $\hat{\phi}$ direction (cylindrical coordinates).

i. Determine the field B as a function of z and show that $\frac{\partial B}{\partial z}$ is zero at the point midway between the two loops. (10)

ii. If d is properly chosen, the second derivative of B with respect to z also vanishes at the midpoint. Determine the value of d such that the second derivative also vanishes. Note: This is the Helmholtz coil arrangement. (5)

Note:

$$\int \frac{x dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

Question 4: Magnetostatics II

Consider a uniformly charged solid sphere of total charge Q and radius a. The sphere is spinning around an axis through its center with a constant angular velocity ω.

(a) It can be shown that the vector potential for a shell of radius R with uniform surface charge density σ is given by (15)

$$\mathbf{A}_{shell} = \frac{\mu_0 \omega \sigma R^4}{3} \frac{\sin\theta}{r^2} \hat{\phi}.$$

Use this result together with the superposition principle to show that the vector potential for the sphere is given by

$$\mathbf{A} = \frac{\mu_0 \omega Q a^2}{20\pi} \frac{\sin\theta}{r^2} \hat{\phi}.$$

Hint: use the fact that $\sigma = \rho dR$, then calculate ρ.

(b) Calculate the magnetic field $\mathbf{B}$ outside the sphere. (10)

Question 5: Electrodynamics and Alternating Current Circuits .....

(a) Can a battery be used as a primary voltage source in a transformer? Explain your answer. (3)

(b) A capacitor with capacitance C is charged up to a potential V and connected to an inductor with inductance L. At time $t = 0$ a switch is closed.

i. Find the current in the circuit as a function of time. (8)

ii. How does the answer change if a resistor R is included in series with C and L? (10)

(c) When a resistor is connected to a voltage source as the only load, there is no phase difference between voltage and current.

i. What is the phase difference in a circuit with a purely capacitive load? (2)

ii. What is the phase difference in a circuit with a purely inductive load? (2)

Note: To solve an ordinary differential equation of the form

$$\frac{d^2y}{dt^2} + ay = 0,$$

assume the solution is of the form $y = y_0 \exp(-bt)$, then find y_0 and b.