UNIVERSITY OF ESWATINI

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FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

RE-SIT/SUPPLEMENTARY EXAMINATION: 2019/2020

TITLE OF PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: PHY221/P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Useful Mathematical Relations

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Gradient Theorem

$$\int_{\vec{a}}^{\vec{b}} (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$$

Divergence Theorem

$$\int \nabla \cdot \vec{A} d\tau = \oint \vec{A} \cdot d\vec{a}$$

Curl Theorem

$$\int (\nabla \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

Line and Volume Elements

Cartesian: $d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$, $d\tau = dxdydz$

Cylindrical: $d\vec{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$, $d\tau = sdsd\phi dz$

Spherical: $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$, $d\tau = r^2 \sin\theta dr d\theta d\phi$

Gradient and Divergence in Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Dirac Delta Function

$$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$$

Question 1: Electrostatics...36

(a) Given four charges of magnitude q placed on the corners of a square of side a, show that the electric field is zero at the center of the square. (5)

(b) Given three charges of magnitude q placed at the corners of an equilateral triangle of side a show that the electric field is zero at the center of the triangle. (6)

(c) Calculate the electric field at a point z above the center of the square described in part (a). (6)

(d) How much energy would be required to assemble the charges in the configuration described in part (a)? (5)

(e) How much energy would be required to assemble the charges in the configuration described in part (b)? (3)

Question 2: Electrostatic II 37

Consider a sphere of radius R carrying charge whose distribution is $\rho(r) = kr$, where k is a constant. Let the total charge on the sphere be Q.

(a) Determine k in terms of Q and R. (5)

(b) Calculate the electric field inside and outside the sphere. (10)

(c) Calculate the electric potential inside and outside the sphere. (10)

Question 3: Magneto-statics 38

(a) A circular loop of radius a lies in the xy plane with the origin at its center.

i. Use the Biot-Sarvat law to find the magnetic field at any point a distance $z > 0$ above the center of the square, i.e. along the z-axis, when a current I circulates counter clockwise around the loop. (8)

ii. Show that $B = \mu_0 I/2a$ at the center of the loop, i.e. at the origin. (2)

(b) Show that if a charge moves a distance $d\vec{l}$ the work done by magnetic forces is zero. (5)

(c) A square of side $2a$ lies in the xy plane with the origin at the center. The sides of the square are parallel to the axes, and a current I goes around it in a counterclockwise sense as seen from a positive value of z (i.e. in the $\hat{\phi}$ direction).

i. Find **A** at the origin. (8)

ii. What is the $\nabla \cdot \mathbf{A}$ at the origin? (2)

Note:

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln |x + \sqrt{a^2 + x^2}| + C$$

Question 4: Magnetostatcs II 39

(a) Give the full definition of a steady current, i.e also state the properties of a steady current. (4)

(b) State the differential and integral forms of Ampere's law. (2)

(c) Briefly describe the mechanisms responsible for paramagnetism and diamagnetism. (4)

(d) A metal rod of length L moves with velocity v in a direction perpendicular to its axis and to a constant magnetic field B that is out of the page, as shown in Fig. 1.

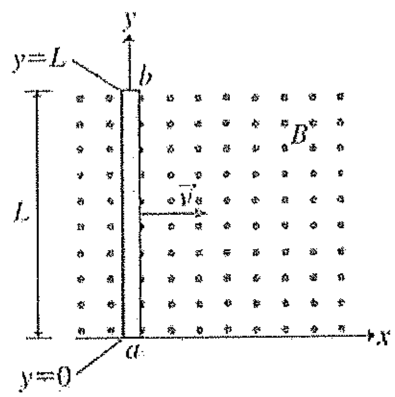

Figure 1: A rod moving in a magnetic field with constant velocity.

i. Write the expression for the force on charges in the rod by virtue of the motion of the rod. (3)

ii. What will be the magnitude and direction of the electric field set up by the resulting separation of charges due to the force? **Note: metals have free electrons.** (4)

iii. What is the potential difference between the ends of the rod? (3)

iv. If the rod moves on a stationary conducting loop as shown in Fig. 2, what current will flow in the loop if the resistance is R? (2)

v. Calculate the emf induced by the motion by means of Faraday's law of induction and compare with part (c). 40 (3)

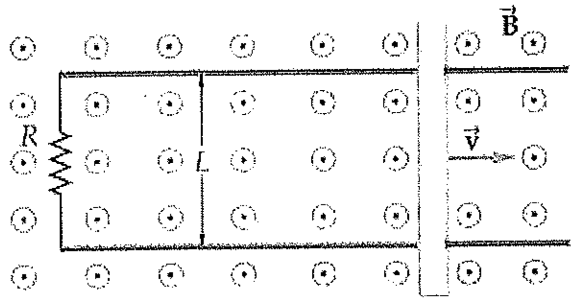

Figure 2: A rod moving over a stationary loop, inside a constant magnetic field.

Question 5: Circuits and Electrodynamics 41

Imagine charging up a capacitor by connecting it to a battery and a resistor, with the battery's voltage fixed at V_0.

(a) Determine the charge $Q(t)$ and current $I(t)$ as functions of time. (8)

(b) Find the total energy of the battery. (5)

(c) Determine the heat delivered to the resistor. (5)

(d) What is the final energy stored in the capacitor? (5)

(e) What fraction of the work done by the battery shows up as energy in the capacitor? (2)