UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

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SUPPLEMENTARY EXAMINATION 2019

TITLE OF PAPER: STATISTICAL PHYSICS AND THERMODYNAMICS

COURSE NUMBER: PHY461

TIME ALLOWED: THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

APPENDICES 1 AND 2 CONTAIN DEFINITE INTEGRALS AND PHYSICAL CONSTANTS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION ONE

(a) In statistical thermodynamics, particles exist in phase space.

(i) Briefly explain what is meant by **phase space**. (2 marks)

(ii) Draw a clearly labelled simple axis illustration of a volume element in phase space for x, y, z and p_x, p_y, p_z coordinates. (1 mark)

(b) Briefly explain the following terms:

(i) **Statistical weight** (2 marks)

(ii) **Degeneracy** (2 marks)

(c) Four coins marked a, b, c and d are tossed. The number of heads (H) and the number of tails (T) obtained in a toss define a macrostate.

(i) Write down all the possible macrostates. (2 marks)

(ii) Find the number of microstates corresponding to each of the above macrostates using the formula:

$$W = \frac{N!}{\prod_s n_s!}$$

(4 marks)

(iii) What is the most probable configuration of the system? Explain your reasoning. (2 marks)

(d) Derive the following Maxwell-Boltzmann distribution function for a system of classical particles in thermal equilibrium:

$$n_s = g_s \, e^{\alpha+\beta\varepsilon_s},$$

where the symbols have their usual meanings. (10 marks)

QUESTION TWO

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(a) Briefly explain the relationship between **entropy** and **thermodynamic probability**?

(3 marks)

(b) Show that the entropy of a system is related to the weight by the following equation:

$$S = k \ln W$$

(8 marks)

(c) (i) State the **equipartition** theorem. (2 marks)

(ii) What does the **partition function** tell us about a system of particles? (2 marks)

(iii) Briefly explain the **Gibb's paradox**. (2 marks)

(d) Consider a particle having energy due to its motion in the x direction being made up of quadratic terms in position x, that is $E_x = ax^2$. Show that

$$\overline{E_{xtot}} = \frac{1}{2}kT$$

(8 marks)

QUESTION THREE

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(a) Briefly explain the following:

(i) The difference between **extensive** and **intensive** variables and give examples in each case. (4 marks)

(ii) Non-interacting or weakly interacting particles. (2 marks)

(iii) An isolated system of particles. (2 marks)

(b) Show that for a classical gas, the specific heat is given by the following expression:

$$C_V = Nk\left(2T\frac{\partial \ln Z}{\partial T} + T^2\frac{\partial^2 \ln Z}{\partial T}\right)$$

(4 marks)

(c) Given the density of states of a system of particles:

$$g(\varepsilon)\,d\varepsilon = \frac{2\pi}{h^3}\,V\,(2m)^{\frac{3}{2}}\,\varepsilon^{\frac{1}{2}}\,d\varepsilon$$

and that

$$Z = \sum_s g_s e^{\beta\varepsilon_s}$$

(i) Show that for a classical perfect gas, the partition function is given by:

$$Z = \frac{V}{h^3}(2\pi mkT)^{\frac{3}{2}}$$

(ii) Show that for classical perfect gas, the total energy is given by: (7 marks)

$$E = \frac{3}{2}NkT$$

(6 marks)

QUESTION FOUR

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(a) Briefly explain the following terms associated with a system of bosons:

(i) **Bose-Einstein condensation** (2 marks)

(ii) **Stefan-Boltzmann law** (2 marks)

(b) State the difference between **photons** and **phonons**.

(2 marks)

(c) Helium 4 (^{4}He) obeys Bose-Einstein statistics, however, as it cools down it exhibits interesting behaviour. Briefly explain the behaviour of ^{4}He as it is cooled down.

(4 marks)

(d) Given that a simple harmonic oscillator can exist only in any of the discrete energy states with energy:

$$\varepsilon = \left(n + \frac{1}{2}\right) h\nu$$

Derive the expression for its mean energy, given that:

$$\epsilon = kT^2 \frac{\partial}{\partial T} \text{ln} Z$$

(10 marks)

(e) (i) Sketch a grath showing the variation of C_V with temperature of an insulator according to Einstein's theory. (3 marks)

(ii) Explain the graph in (d)(i) with reference to the classical Dulong-Petit law. (2 marks)

QUESTION FIVE

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(a) Given the Fermi function for a system of fermions as:

$$f(\varepsilon_s) = \frac{1}{e^{\frac{(\varepsilon_S - \varepsilon_F)}{kT}} + 1}$$

Evaluate $f(\varepsilon)$ for the following cases:

(i) $\varepsilon > \varepsilon_F(0)$.

(2 marks)

(ii) $\varepsilon < \varepsilon_F(0)$.

(2 marks)

(iii) Explain in detail the results obtained in (a)(i) and (a)(ii).

(4 marks)

(b) The Fermi level of a metal is -6.8 eV.

(i) Find the probability that an electron can have energies 0.1 eV and 1.0 eV above the Fermi level at T = 300 K and T= 400 K. (6 marks)

(ii) Comment on the results obtained in (b)(i). (2 marks)

(c) The Fermi energy of such a system of fermions is given by:

$$\varepsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{\frac{2}{3}}$$

(i) Define **Fermi energy**.

(2 marks)

(ii) Calculate the Fermi energy of a metal, which has a density of $8.6 \times 10^2\ kg.m^{-3}$ and atomic weight of 39. Give your answer in electron volts.

(5 marks)

(iii) Considering that $T_F = \varepsilon_F / k$. Calculate the Fermi temperature for the metal in (a)(i).

(2 marks)

APPENDIX 1

Useful definite integrals

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$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2} x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2} x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2} x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2} x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{\frac{1}{2}}$$

$$\int_0^\infty e^{-ax^2} x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty e^{-\lambda x} x^{\frac{1}{2}}\,dx = \frac{\pi^{\frac{1}{2}}}{2\lambda^{\frac{3}{2}}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{\frac{1}{2}}}{e^x - 1}\,dx = \frac{2.61\pi^{\frac{1}{2}}}{2}$$

APPENDIX 2

Table of physical constants

QUANTITY	SYMBOL	VALUE
Speed of light	c	3.00×10^{8} m.s^{-1}
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} J.K^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J.mol^{-1}.K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} J.T^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H.m^{-1}
Stefan constant	σ	5.67×10^{-8} W.m^{-2}.K^{-4}
Atmospheric pressure		1.01×10^{5} N.m^{-2}
Mass of $^{4}_{2}$He atom		6.65×10^{-27} kg
Mass of $^{3}_{2}$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.41 mol^{-1}