UNIVESITY OF ESWATINI
FACULTY OF SCIENCE AND ENGINEERING
DEPARTMENT OF PHYSICS

Examination 2019/2020

COURSE NAME: ADVANCED COMPUTATIONAL PHYSICS
COURSE CODE: PHY601
TIME ALLOWED: 3 hours

Answer All Questions in Section A. Choose **two (2)** Questions in section B.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Question 1

(a) Match up the following three numerical differentiation schemes, approximating the solution to the ordinary differential equation $x'(t) = F(x(t))$, with their correct name (Runge-Kutta, Euler or Picard).

A. $X_{n+1} = x_n + F(x_n) \triangle t$

B. $X_{n+1} = x_n + \frac{1}{2}[F(x_n) + F(x_{n+1})] \triangle t$

C. $X_{n+1} = x_n + F\left(x_n + \frac{1}{2}F(x_n) \triangle t\right) \triangle t$

[6 marks]

(b) Consider the finite difference approximation

$$f'(a) = \frac{-3f(a) + 4f(a+h) - f(a+2h)}{2h} + O(h^p).$$

What is the value of p?

[2 marks]

(c) Derive the fourth-order Runge–Kutta algorithm for solving the differential equation

$$\frac{dy}{dt} = g(y,t)$$

with a given initial condition. Discuss the options in the selection of the parameters involved.

[5 marks]

Question 2

(a) The simple program in figure 1 is written to read in the radius and calculate the area of the corresponding circle and volume of the sphere.

(i) Modify this program such that it stops with a warning if the radius given is negative.

[5 marks]

```
! ****************************************************
program Cirle
! ****************************************************
implicit none
real*8 PI,r,A,V

  PI=4.0d0*atan(1.0d0)
  write(*,*) 'please enter the value of thr radius,r'
  read(*,*) r

   A=PI*r**2
   V=4*PI*r**3
 write(*,*) 'The area is,', A
 write(*,*) 'The Volume is,', V
end
!
```

Figure 1: A program for calculating the area and volume, given the radius.

(ii) Modify the program such that it computes a complex value of the volume and area, given a complex value of the radius.

[10 marks]

(b) Which of the following statements in figure 2 are incorrect declarations and why? (If you think a declaration may be correct in a given situation then say what the situation would be.)

[22 marks]

(c) Given
`INTEGER::i=3, j=7`
`REAL, DIMENSION(1:20) ::A`
Which of the following are valid array references for the array

- `A(12)`
- `A(21)`
- `A(I)`
- `A(3.0)`
- `A(I*J)`
- `A(I*INT(4.0*ATAN(1.0)))`

[10 marks]

```
1.ReAl :: x
2.CHARACTER :: name
3.CHARACTER(LEN=10) :: name
4.REAL :: var-1
5.INTEGER :: 1a
6.CHARACTER(LEN=5) :: town = "Glasgow"
7.CHARACTER(LEN=*) :: town = "Glasgow"
8.CHARACTER(LEN=*), PARAMETER :: city = "Glasgow"
9.REAL :: pi = +22/7
10.LOGICAL :: wibble = .TRUE.
11.CHARACTER(LEN=*), PARAMETER :: "Bognor"
12.REAL, PARAMETER :: pye = 22.0/7.0
13.REAL :: two_pie = pye*2
14.REAL :: a = 1., b = 2
15.LOGICAL(LEN=12) :: frisnet
16.CHARACTER(LEN=6) :: you_know = 'y'know"
17.CHARACTER(LEN=6) :: you_know = "y'know"
18.INTEGER :: ia ib ic id in free format source form
19.DOUBLE PRECISION :: pattie = +1.0D0
20.DOUBLE PRECISION :: pattie = -1.0E-0
21.LOGICAL, DIMENSION(2) bool
22.REAL :: poie = 4.*atan(1.)
```

Figure 2:

Section B

1. Consider the Poisson equation

$$\nabla^2\phi(x,y) = -\frac{\rho(x,y)}{\epsilon_0}$$

from electrostatics on a rectangular geometry with $x \in [o, L_x]$ and $x \in [o, L_y]$. Write a program that solves this equation using the relaxation method. Test you program with

(a) $\rho(x,y) = 0$, $\phi(0,y) = \phi(L_x,y) = \phi(x,0) = 0$, $\phi(x,L_y) = 1V$, $L_x = 1m$ and $L_y = 1.5m$

(b) $\frac{\rho(x,y)}{\epsilon_0} = 1V/m^2$, $\phi(0,y) = \phi(L_x,y) = \phi(x,0) = \phi(x,L_y) = 0$, $L_x = L_y = 1m$

[20 marks]

2. A point mass that can move along a straight line is attached to an end of an ideal elastic spring. (The other end of the spring is fixed.) A

viscous friction force proportional to the cube of the velocity is acting on the mass. Therefore, the motion of the particle is described by the following differential equation;

$$\ddot{x} + \mu \dot{x}^3 + x = 0,$$

where μ is a positive dimensionless coefficient of nonlinear friction. (The equation is written in dimensionless units). Write a program that solves this equation numerically for the following conditions;

- $\mu = 0.2$, $x(0) = 1$, $\dot{x}(0) = 0$, plot $x(t)$ and $\dot{x}(t)$ for $0 < t < 50$.

[20 marks]

3. The temperature distribution $u(x,t)$ in a thin rod satisfies the equation

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}$$

together with the boundary conditions

$$u(0,t) = u(1,t) = 0$$

at the ends $x = 0, 1$. The initial temperature distribution of $t = 0$ is given by the function

$$u(x,0) = \begin{cases} 0.5, & \text{if } x \in [x_1, x_2] \\ 0.3, & \text{if } x \notin [x_1, x_2], \end{cases}$$

where $x_1 = 0.25$ and $x_2 = 0.75$. Write a program that calculates the temperatire distribution $u(x, t_f)$ for $t_f = 0.0001, 0.001, 0.01, 0.05$. Take $N_x = 100$ and $N_t = 1000$. Do the same for $t_f = 0.1$ by choosing the appropriate N_x and keeping $N_t = 1000$. Plot the functions $u(x, t_f)$ in the same plot.

[20 marks]