UNIVERSITY OF ESWATINI

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MAIN EXAMINATION: FRIDAY, 14^{th} AUGUST, 2020

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF PHYSICS

COURSE NAME : $\langle Q|UANTUM\ COMPUTIN|G\rangle$

COURSE CODE : $|PHY606\rangle$

ALLOCATED TIME : THREE (3) HOURS

INSTRUCTIONS:

- THIS EXAMINATION HAS THREE (3) QUESTIONS. ANSWER ALL QUESTIONS
- POINTS FOR DIFFERENT SECTIONS ARE INDICATED IN THE RIGHT-HAND MARGIN
- THE TOTAL NUMBER OF MARKS IS 100.

THE PAPER HAS 5 PAGES, INCLUDING THIS PAGE.

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Question 1

The Pauli matrices are defined as follows:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(a) Consider the two quantum states:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle,$$

$$|\phi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

Compute the following:

(i) $|\psi\rangle \otimes |\psi\rangle$

[3 marks]

(ii) $\langle\psi| \otimes \langle\psi|$

[3 marks]

(iii) $(\sigma_x \otimes \mathcal{I})(|\psi\rangle \otimes |\psi\rangle)$

[4 marks]

(iv) $(\sigma_x \otimes \sigma_x)(|\psi\rangle \otimes |\psi\rangle)$

[4 marks]

(b) Suppose we wanted to measure the operator $\hat{n} \cdot \vec{\sigma}$ on a qubit that is initially in the state $|0\rangle$.

(i) What are the corresponding measurement operators?

[12 marks]

(ii) What is the probabilty of obtaining the result +1, in terms of spherical polar angles θ and ϕ?

[6 marks]

(iii) What is the state after measurement?

[5 marks]

Question 2

(a) Consider Grover's search algorithm and assume we have M target states out of a total N states. The black box O takes:

$$O|x\rangle = \begin{cases} -|x\rangle & \text{if } x \text{ is a target state;} \\ |x\rangle & \text{otherwise.} \end{cases}$$

Now suppose we find a target state with probability 1 after one iteration of the algorithm. What can be said about the ratio $\frac{M}{N}$?

[8 marks]

(b) Now consider a modification to Grover's algorithm so that the oracle now performs:

$$O|x\rangle = \begin{cases} e^{i\phi}|x\rangle & \text{if } x \text{ is a target state;} \\ |x\rangle & \text{otherwise.} \end{cases}$$

Show that if the following transformation is used:

$$G = H^{\otimes n}\left[(1 - e^{i\phi}|0\rangle\langle 0| - I)\right] H^{\otimes n} O,$$

instead of the standard Grover iteration, you can always choose ϕ such that the algorithm finds a target state with probability 1 after one iteration, for any state with $\frac{M}{N}$ sufficiently large. For what values of $\frac{M}{N}$ is there such a ϕ?

[18 marks]

Question 3

(a) What is the density matrix if you have a qubit which is in state:

$$|0\rangle \text{ with probability } \frac{1}{3},$$

$$\frac{-1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle \text{ with probability } \frac{1}{3},$$

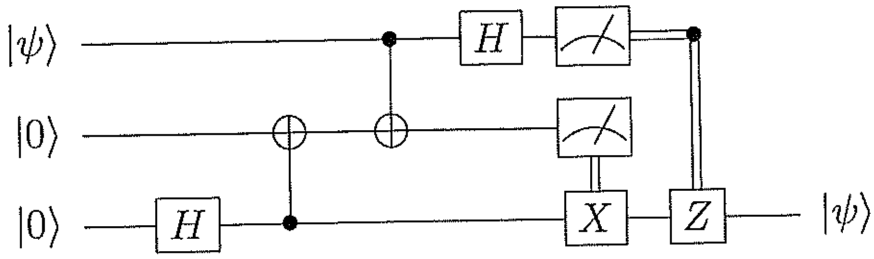
$$\frac{-1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle \text{ with probability } \frac{1}{3}.$$

[8 marks]

(b) In the teleportation protocol, show that the probability distribution for the values of the two qubits that Alice sends to Bob is independent of the state ψ of the qubit being transmitted.

[12 marks]

(c) Work out the details of the following quantum circuit which moves a state from qubit one (1) to qubit three (3). Given the possible states, 0 or 1, in qubits one and two, summarize all the possible scenarios in a table which looks like the template given below:



Qubit 1	Qubit 2	State of qubit 3	Correction step	Final state
0	0			
0	1			
1	0			
1	1			

[17 marks]

!!!!!!!!THIS IS THE END OF THE EXAMINATION PAPER!!!!!!!!