UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2020/2021

COURSE NAME: SOLID STATE PHYSICS

COURSE CODE: PHY412

TIME ALLOWED: 3 HOURS

INSTRUCTIONS

- There are five questions in this paper, and each question carries a total of 25 marks.
- Answer any four questions in your preferred order.
- Points for different questions are shown in the right-hand margins.
- Additional materials included in this paper are a list of fundamental constants in Physics.

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Question One [25 Marks]

The total energy per molecule of an ionic crystal can be written as a function of the interionic separation r as follows:

$$U_{\text{ion}} = \frac{\alpha \nu e^2}{4\pi\epsilon_0 r} + B \exp\left(-\frac{r}{\rho}\right) .$$

(a) Briefly describe the parameters α, ν and ρ. [6]

(b) Show that the cohesive energy per molecule of the crystal can be expressed as:

$$U_{\text{ion}}(r_0) = \frac{\alpha \nu e^2}{4\pi\epsilon_0 r_0}\left(1 - \frac{\rho}{r_0}\right) ,$$

where r_0 is the equilibrium separation between two nearest atoms. [12]

(c) Show that stability of the ionic crystal can only occur for the condition $r_0 < 2\rho$. [7]

Question Two [25 Marks]

(a) Define the folllowing terms/phrases concerning crystal structures:

(i) Lattice [1]

(ii) Basis [1]

(iii) Primitive unit cell [1]

(iv) Wigner-Seitz primitive cell [1]

(v) Bravais lattice [1]

(b) Aluminum(Al), whose atomic mass is 26.982 u, forms an FCC structure and has an atomic radius of 0.143 nm.

(i) Calculate the lattice constant, a. [3]

(ii) Calculate the atomic packing factor, PF. [3]

(iii) Calculate the theoretical density of the metal in the unit cell. How does this compare to the actual density of aluminum ($2.700 g/cm^3$)? Why is there a difference? [6]

(c) Derive the following formula for the general interplanar spacing for crystalline systems with orthogonal axes:

$$d_{hk\ell} = \left(\frac{h^2}{a_1^2} + \frac{k^2}{a_2^2} + \frac{\ell^2}{a_3^2}\right)^{-1/2}.$$

[8]

Question Three [25 Marks]

Consider a one-dimensional chain of vibrating identical atoms of mass m, and suppose a is the equilibrium separation between two adjacent atoms and K is the spring constant for the elastic bonds connecting nearest neighbors.

(a) By starting from the equation of motion for the n^{th} atom and treating the displacement as a plane wave with wave-vector $\boldsymbol{k}$, derive the following formula for the dispersion relation:

$$\omega(k) = \sqrt{\frac{4K}{m}}\left|\sin\left(\frac{ka}{2}\right)\right| .$$

[12]

(b) Sketch the dispersion relation in the first and second Brillouin zones. Mark clearly the boundaries of the Brillouin zones. [4]

(c) Find expressions for the phase velocity and group velocity. [6]

(d) Briefly discuss the non-dispersive (acoustic wave) regime of the dispersion relation. [3]

Question Four [25 Marks]

(a) Diffraction studies involving X-rays or electrons give information about the crystallographic properties of solids.

  (i) Compare these two techniques with reference to particle energies and type of information that can be obtained. [4]

  (ii) Which technique is most appropriate for studying surface crystallography? [1]

(b) State five assumptions of the Drude Model for electron transport properties in metals. [5]

(c) Discuss one major success and one major failure of the Drude Model. [2]

(d) Starting from the equation of motion of an electron in an electric field, show that the electric conductivity is

$$\sigma = \frac{ne^2\tau}{m_e},$$

where n is the number of electrons, τ is the relaxation time between collisions, and the other symbols have their usual meaning. [10]

(e) The resistivity of copper is $1.7 \times 10^{-6}\,\Omega$cm and the atomic density of copper is 8.5×10^{22} atoms/cm^3. Estimate the collision time τ for an electron in copper. [3]

Question Five [25 Marks]

(a) The Fermi energy for copper at temperature $T = 300\,\mathrm{K}$ is $7.0\,\mathrm{eV}$. The electrons in copper follow the Fermi-Dirac distribution function. Find the probability of an energy level at $7.15\,\mathrm{eV}$ being occupied by an electron. [3]

(b) Metallic sodium crystallizes in body-centered cubic form, the length of the cube being $4.25 \times 10^{-8}\,\mathrm{cm}$.

(i) Find the concentration, n, of conducting electrons. [3]

(ii) Adopting the free-electron Fermi gas model for the conduction electrons, derive the following expression for the Fermi energy (at 0K):

$$E_F = \frac{\hbar^2}{2m_e}(3\pi^2 n)^{\frac{2}{3}} .$$

Hence show that it depends only on the concentration of conducting electrons, but not on the mass of the metal. [8]

(iii) Calculate the value of E_F for metallic sodium. [3]

(iv) Provide expressions for the Fermi temperature and the Fermi velocity in terms of E_F. [4]

(c) Assuming that the energy dispersion of a band in a semiconductor can be expressed as $E = Ak^2$, where $A = 84.67\,\mathrm{Å}^2 \cdot \mathrm{eV}$, calculate the electron effective mass in this band, in units of free electron rest mass m_0. [4]

END OF QUESTIONS

Physical Constants

Constant	Symbol	Value
Avogadro's number	N_A	$6.022 \times 10^{23}\,\mathrm{mol^{-1}}$
Elementary charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Electron rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg}$
Neutron rest mass	m_n	$1.675 \times 10^{-27}\,\mathrm{kg}$
Atomic mass unit (dalton)	amu (Da)	$1.661 \times 10^{-27}\,\mathrm{kg}$
Gas constant	R	$8.315\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Boltzmann's constant	$k = R/N_A$	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Standard acceleration of gravity	g	$9.807\,\mathrm{m\,s^{-2}}$
Speed of light	c_0	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Faraday constant	$F = eN_A$	$9.649 \times 10^{4}\,\mathrm{C\,mol^{-1}}$
Permeability of the vacuum	μ_0	$4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$ (exact)
Permittivity of the vacuum	$\varepsilon_0 = 1/\mu_0 c_0^2$	$8.854 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$

Conversion Factors for Non-SI Units

1 dyne = 10^{-5} N
1 bar = 10^{5} Pa
1 atm = 1.013 bar
1 torr = 1/760 atm
1 psi = 6.895×10^{3} Pa
1 int. cal = 4.187 J
1 erg = 10^{-7} J

1 eV = 1.602×10^{-19} J
1 eV/particle = 96.48 kJ $\mathrm{mol^{-1}}$
1 D = 3.336×10^{-30} C m
1 l = 1 $\mathrm{dm^3}$ = $10^{-3}\,\mathrm{m^3}$
1 $\mathrm{cm^{-1}}$ = 1.986×10^{-23} J
hc/k = 1.438 cm K

Prefixes

pico p = 10^{-12}
nano n = 10^{-9}
micro μ = 10^{-6}
milli m = 10^{-3}

kilo k = 10^{3}
mega M = 10^{6}
giga G = 10^{9}
tera T = 10^{12}