UNIVERSITY OF SWAZILAND
FACULTY OF SOCIAL SCIENCE
DEPARTMENT OF ECONOMICS

MAIN EXAMINATION PAPER: MAY 2007

TITLE OF PAPER: QUANTITATIVE METHODS
COURSE CODE: ECON 205
TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS:

1. Answer Two Questions from Each Section to Make a Total of Four

2. Show all relevant workings to your answer

3. All Questions carry a total of 25 marks

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATOR

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR

SECTION A

Question 1

a) Given the following macroeconomic model:

$$Y = C + I + G + X - M$$
$$C = c_0 + c_1 Y$$
$$M = m_0 + m_1 Y$$

Where; Y = Income, C = Consumption, $I = I_0$ = Investment,
$G = G_0$ = Government Expenditure, $X = X_0$ = Exports, M = Imports

Determine expressions for the following

i)	Equilibrium income	[5 marks]
ii)	Consumption at equilibrium	[3 marks]
iii)	The level of imports at equilibrium	[3 marks]

b) It is known in the university bookshop that when the price of a certain book is E10 there will be no purchases, but it is thought that for every Lilangeni drop in price, twelve new purchases would appear. Furthermore it is known that the publishers refuse to offer any copies for sale at a price of E3 or less, but they are prepared to offer 9 copies for E4 and an additional nine copies for each one Lilangeni rise in price. Find two equations showing:

i) the quantity demanded (Q_d) in terms of price.   [5 marks]
ii) the quantity supplied (Q_s) in terms of price.   [5 marks]

c) Find the equilibrium price at which quantity demanded will equal the quantity supplied.   [4 marks]

2

Question 2

a) Let the production function for a firm be as specified below:

$$Q = 4LK - 3L^2 - 2K^2 + 6L + 14K$$

Where Q is the output of the firm, L and K are the numbers of units of labour and capital respectively that the firm chooses to hire.

i) If the firm is a perfect competitive producer and sells its product at E25 per unit, what are the profit maximizing levels of L and K? The firm currently pays E7 per unit of labour and E14 per unit of capital.
   [9 marks]

b) if the marginal cost curve of the firm is given as:

$$MC = 100 - 4Q + Q^2$$

What is the total cost of increasing output from Q = 20 to Q = 30?   [8 marks]

Question 3

a) Use Cramer's rule to solve for X and Y given the first order conditions for constrained optimization as follows:

$$\delta TC/\delta X = 16x - y - \lambda = 0$$
$$\delta TC/\delta Y = 24y - x - \lambda = 0$$
$$\delta TC/\delta \lambda = 42 - x - y = 0$$

[10 marks]

b) South Africa, Mozambique, and Lesotho have agreed on an economic partnership that will foster the respective countries' development. South Africa has great expertise in High-Tech goods and wishes to export these to Mozambique and Lesotho. Mozambique has large off-shore resources of gas which it wishes to export to South Africa and Lesotho. Lesotho which is mountainous has recently completed a large Hydro electric scheme at its Kgasi dam and wishes to export its electric power to the other two countries, provided an agreement over a grid system can be reached.

3

The Ministers of Economic planning and Development have agreed that to produce one unit of High-Tech product, 0.2 units of the product, 0.2 units of gas, and 0.2 units of electricity are required. For each unit of Gas, 0.4 units of High-Tech, 0.2 units of Gas, and 0.1 units of electricity are required. For each unit of electricity, 0.4 units of High-Tech, 0.2 units of Gas and 0.1 units of electricity are required. Furthermore final demand is anticipated (in appropriate units) for the three products to be in the coming year:

	HIGH-TECH	GAS	ELECTRICITY
South Africa	500	2000	1500
Mozambique	750	2500	500
Lesotho	1000	500	3000

i) Formulate the matrix representation of the input-output model associated with this problem.   [5 marks]
ii) Calculate the level of output for the three countries.   [15 marks]

Question 4

Write explanatory notes on the following concepts

   a) Symmetric matrix and singular matrix
   b) Endogenous variables and exogenous variables
   c) Binding constraints and non-negativity constraints
   d) The meaning of equilibrium in economic modelling
   e) The meaning of optimization in economics   [5 marks each]

4

SECTION B

Question 5

a) An investor has funds in Standard Bank and Barclays Bank. He assesses the probability that in five years time, Standard Bank will fail at 0.0001 and assigns the same failure probability to Barclays. Further, this businessman thinks the probability that both banks will fail is 0.00001.

i) What is the probability that Standard or Barclays Bank will fail?

[5 marks]

ii) Are the events Standard bank fails and Barclays Bank fails independent? Why?   [5 marks]

iii) What is the probability that Barclays fails if Standard bank fails? [5 marks]

b) The expected lifetime of electric light bulbs was 1500hrs with standard deviation of 90hrs. To test a new batch a sample of 100 showed a mean lifetime of 1480 hours. Test the hypothesis that the mean lifetime of the electric light bulbs has not changed, using a significance of 0.05.

[5 marks]

c) What are the four interpretations of probability?   [5 marks]

Question 6

Consider the following data on household expenditure, Y, and its determinants, X and W:

Y	X	W
23	3	6
3	7	2
19	5	7
15	3	2
4	8	4
7	5	1
11	7	6
22	2	4

a) Determine the regression equation $Y = a + bX + cW$ [13 marks]
b) Calculate the standard error of estimate [4 marks]
c) Calculate the multiple coefficient of determination and interpret it.
[4 marks]
d) Interpret the values a, b, and c. [4 marks]

Question 7

A farmer owns a 100-acre farm and plans to plant at most three crops. The seed for crops A, B, and C cost E40, E20, and E30 per acre, respectively. A maximum of E3, 200 can be spent on seed. Crops a, B, and C respectively require 1, 2, and 1 workdays per acre, respectively, and there are a maximum of 160 workdays available. The farmer can make a profit of E100 per acre on crop A, E300 per acre on crop B, and E200 per acre on crop C.

i) Formulate a linear programming problem that will represent the above relationships.

[10 marks]

ii) Use the Simplex method to determine the amount of acres of each crop that should be planted to maximize profit.

[15 marks]

Question 8

Swaziland Electricity Board (SEB) ha two generating schemes in operation, scheme A and scheme B. The total cost function for each scheme is given by:

$$TC_A = 12 + 7Q_A - 2Q^2_A + 0.75Q^3_A$$

$$TC_B = 5 + 3Q_B + 1.5Q^2_B$$

Where TC_A and TC_B are the total costs, Q_A and Q_B are kilowatt hours (in thousand of units) generated under each scheme. The company wishes to minimize the cost of generating any given amount of electricity. Using the Lagrange multiplier method:

a) Determine how should it allocate production between the two generating schemes if it must produce 20 000 kilowatt hours?   [18 marks]

b) Establish the total cost of the firm?   [7 marks]

1. *Areas in the right-hand tail of the normal distribution*

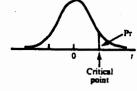| z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00315									
3.1	.00097									
3.2	.00069									
3.3	.00048									
3.4	.00034									
3.5	.00023									
3.6	.00016									
3.7	.00011									
3.8	.00007									
3.9	.00005									
4.0	.00003									

3. *Student's t Critical Points*

d.f. \ Pr	.25	.10	.05	.025	.010	.005	.001
1	1.000	3.078	6.314	12.706	31.821	43.657	318.31
2	.816	1.886	2.920	4.303	6.965	9.925	22.326
3	.765	1.638	2.353	3.182	4.541	5.841	10.213
4	.741	1.533	2.132	2.776	3.747	4.604	7.173
5	.727	1.476	2.015	2.571	3.365	4.032	5.893
6	.718	1.440	1.943	2.447	3.143	3.707	5.208
7	.711	1.415	1.895	2.365	2.998	3.499	4.785
8	.706	1.397	1.860	2.306	2.896	3.355	4.501
9	.703	1.383	1.833	2.262	2.821	3.250	4.297
10	.700	1.372	1.812	2.228	2.764	3.169	4.144
11	.697	1.363	1.796	2.201	2.718	3.106	4.025
12	.695	1.356	1.782	2.179	2.681	3.055	3.930
13	.694	1.350	1.771	2.160	2.650	3.012	3.852
14	.692	1.345	1.761	2.145	2.624	2.977	3.787
15	.691	1.341	1.753	2.131	2.602	2.947	3.733
16	.690	1.337	1.746	2.120	2.583	2.921	3.686
17	.689	1.333	1.740	2.110	2.567	2.898	3.646
18	.688	1.330	1.734	2.101	2.552	2.878	3.610
19	.688	1.328	1.729	2.093	2.539	2.861	3.579
20	.687	1.325	1.725	2.086	2.528	2.845	3.552
21	.686	1.323	1.721	2.080	2.518	2.831	3.527
22	.686	1.321	1.717	2.074	2.508	2.819	3.505
23	.685	1.319	1.714	2.069	2.500	2.807	3.485
24	.685	1.318	1.711	2.064	2.492	2.797	3.467
25	.684	1.316	1.708	2.060	2.485	2.787	3.450
26	.684	1.315	1.706	2.056	2.479	2.779	3.435
27	.684	1.314	1.703	2.052	2.473	2.771	3.421
28	.683	1.313	1.701	2.048	2.467	2.763	3.408
29	.683	1.311	1.699	2.045	2.462	2.756	3.396
30	.683	1.310	1.697	2.042	2.457	2.750	3.385
40	.681	1.303	1.684	2.021	2.423	2.704	3.307
60	.679	1.296	1.671	2.000	2.390	2.660	3.232
120	.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	.674	1.282	1.645	1.960	2.326	2.576	3.090

.09
.9139
.8270
.7483
.6771
.6126
.5543
.5016
.4538
.4107
.3716
.3362
.3042
.2753
.2491
.2254
.2039
.1845
.1670
.1511
.1367
.1237
.1119
.1013
.0916
.0829
.0750
.0679
.0614
.0556
.0503
.0455
.0412
.0373
.0337
.0305
.0276
.0250
.0226
.0204
.0185
.09