UNIVERSITY OF SWAZILAND
FACULTY OF SOCIAL SCIENCE
DEPARTMENT OF ECONOMICS

MAIN EXAMINATION PAPER: MAY 2009

TITLE OF PAPER: QUANTITATIVE METHODS
COURSE CODE: ECON 205
TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS:

1. Answer Two Questions from Each Section to Make a Total of Four

2. Show all relevant workings to your answer

3. All Questions carry a total of 25 marks

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATOR

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR

SECTION A

Question 1

A farmer owns a 100-acre farm and plans to plant at most three crops. The seed for crops A, B, and C cost E40, E20, and E30 per acre, respectively. A maximum of E3, 200 can be spent on seed. Crops A, B, and C respectively require 1, 2, and 1 workdays per acre, respectively, and there are a maximum of 160 workdays available. The farmer can make a profit of E100 per acre on crop A, E300 per acre on crop B, and E200 per acre on crop C.

i) Formulate a linear programming problem that will represent the above relationships.

[10 marks]

ii) Use the Simplex method to determine the amount of acres of each crop that should be planted to maximize profit.

[15 marks]

Question 2

(a) If the manufacturer of cut glass has a production given by $Q= 15K^{0.3} L^{0.6,}$ find the marginal product of labour and capital. If the current levels of labour and capital are 12 and 30, respectively, determine the effect on output of an additional unit of labour

[9 marks]

(b) A consumer has a utility function given by:

$$U = \ln(x_1) + 3\ln(x_2)$$

If the budget constraint is given by $10x_1 + 14x_2 = 24$, find the optimum quantities of the two goods that the consumer should purchase in order to maximise utility, subject to the budget constraint. [16 marks]

Question 3

(a) The technological inverse for a three sector economy as well as the planned final demand are given as:

$$(I - A)^{-1} = \begin{pmatrix} 2.4 & 0.6 & 0.3 \\ 2.0 & 3.5 & 2.0 \\ 2.5 & 4.0 & 4.5 \end{pmatrix}$$

$$D = \begin{pmatrix} D1 \\ D2 \\ D3 \end{pmatrix} = \begin{pmatrix} 100 \\ 200 \\ 50 \end{pmatrix}$$

Compute the sectoral total outputs that will enable the economy to realize the planned final demand

[10 marks]

(b) Find the equilibrium income (Y) and rate of interest (r) given the following information on the commodity and money markets of a closed economy without government activities.

I. Commodity Market

(a) Consumption function

$$C = 50 + 2/3\ Y$$

(b) Investment function

$$I = 790 - 21r$$

II. Money market

(a) Transactions and Precautionary demand

$$L_1 = 1/6Y$$

(b) Speculative demand

$$L_2 = 1000 - 9r$$

(c) Money supply

$$M_s = 1200$$

(15 marks)

Question 4

Write explanatory notes on the following concepts

a) An idempotent matrix and a singular matrix
b) Behavioural equation and an identity
c) The objective of linear programming
d) The significance of Break-even analysis
e) The concept of consumer and producer surplus

[5 marks each]

SECTION B

Question 5

a) Write brief explanatory notes on the following

i. Type one and type two error in hypothesis testing
ii. The classical and the logical interpretations of probability
iii. Autocorrelation and heteroscedasticity [5 marks each]

b) What is the significance of the error term (u) in econometric model specicification? State the assumptions surrounding the error term in model specification. [10 marks]

Question 6

a) Give an intuitive explanation of why an econometrician will make the following assumptions when estimating the classical linear regression model

i. Constant error variance
ii. No correlation among errors
iii. Non- stochastic explanatory variables [3 marks each]

b) In a study between the amount of rainfall and the quantity of air pollution removed, the following data were collected:

Daily rainfall, X (0.01 centimeter)	**Particulate removed, Y (micrograms per cubic meter)**
4.3	126
4.5	121
5.9	116
5.6	118
6.1	114
5.2	118
3.8	132
2.1	141
7.5	108

a) plot the data on a scatter diagram and estimate the regression line to predict the particulate removed from the amount of daily rainfall and interpret the coefficients. [10 marks]
b) estimate the amount of particulate removed when daily rainfall (X) is 4.8 units. [2 marks]
c) calculate the product moment correlation coefficient and interpret it [3 marks]
d) determine the percentage variation that is due to changes in the daily amount of rainfall. [1 mark]

Question 7

Consider the following data

	1998		2008	
Commodity	Price (E)	Quantiy	Price(E)	Quantiy
A	1	60	1.50	80
B	2	110	2.30	130
C	3	80	4.20	120
D	4	150	3.60	230

a) Define, calculate, and interpret Laspeyre's Price index
b) Define, calculate, and interpret Fisher's Price index
c) Define, calculate, and interpret Laspeyre's Quantity index
d) Define, calculate, and interpret Fisher's Quantity index
e) Which of the two indexes passes the factor reversal test?
[5 marks each]

Question 8

(a) Among 80 students in 1st year of B.A. Economics course, 48 are men, 22 of the men have O' level mathematics, and 35 of the students altogether have O' level mathematics. One of the students is chosen at random (everyone has an equal chance of selection) to be the representative of the year. What is the probability that:

i. The representative will be a woman without O' level mathematics? [5 marks]

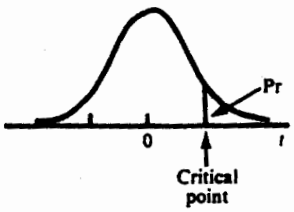
ii. The representative will be woman, given that he/she has O'level mathematics? [5 marks]

iii. The representative will have O'level mathematics, given that representatives are men? [5 marks]

(b) Certain tubes manufactured by a company have a mean lifetime of 800 hours and standard deviation of 60 hours. Find the probability that a random sample of 16 tubes taken from the group will have a mean lifetime between 790 hours and 810 hours. [5 marks]

(c) The expected lifetime of electric light bulbs was 1500 hours with a standard deviation of 90 hours. To test a new batch a sample of 100 showed a mean lifetime of 1480 hours. Test at 5% significance level the hypothesis that the mean lifetime of the electric light bulbs has not changed. [5 marks]

3. *Student's t Critical Points*



d.f. \ Pr	.25	.10	.05	.025	.010	.005	.001
1	1.000	3.078	6.314	12.706	31.821	43.657	318.31
2	.816	1.886	2.920	4.303	6.965	9.925	22.326
3	.765	1.638	2.353	3.182	4.541	5.841	10.213
4	.741	1.533	2.132	2.776	3.747	4.604	7.173
5	.727	1.476	2.015	2.571	3.365	4.032	5.893
6	.718	1.440	1.943	2.447	3.143	3.707	5.208
7	.711	1.415	1.895	2.365	2.998	3.499	4.785
8	.706	1.397	1.860	2.306	2.896	3.355	4.501
9	.703	1.383	1.833	2.262	2.821	3.250	4.297
10	.700	1.372	1.812	2.228	2.764	3.169	4.144
11	.697	1.363	1.796	2.201	2.718	3.106	4.025
12	.695	1.356	1.782	2.179	2.681	3.055	3.930
13	.694	1.350	1.771	2.160	2.650	3.012	3.852
14	.692	1.345	1.761	2.145	2.624	2.977	3.787
15	.691	1.341	1.753	2.131	2.602	2.947	3.733
16	.690	1.337	1.746	2.120	2.583	2.921	3.686
17	.689	1.333	1.740	2.110	2.567	2.898	3.646
18	.688	1.330	1.734	2.101	2.552	2.878	3.610
19	.688	1.328	1.729	2.093	2.539	2.861	3.579
20	.687	1.325	1.725	2.086	2.528	2.845	3.552
21	.686	1.323	1.721	2.080	2.518	2.831	2.527
22	.686	1.321	1.717	2.074	2.508	2.819	3.505
23	.685	1.319	1.714	2.069	2.500	2.807	3.485
24	.685	1.318	1.711	2.064	2.492	2.797	3.467
25	.684	1.316	1.708	2.060	2.485	2.787	3.450
26	.684	1.315	1.706	2.056	2.479	2.779	3.435
27	.684	1.314	1.703	2.052	2.473	2.771	3.421
28	.683	1.313	1.701	2.048	2.467	2.763	3.408
29	.683	1.311	1.699	2.045	2.462	2.756	3.396
30	.683	1.310	1.697	2.042	2.457	2.750	3.385
40	.681	1.303	1.684	2.021	2.423	2.704	3.307
60	.679	1.296	1.671	2.000	2.390	2.660	3.232
120	.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	.674	1.282	1.645	1.960	2.326	2.576	3.090

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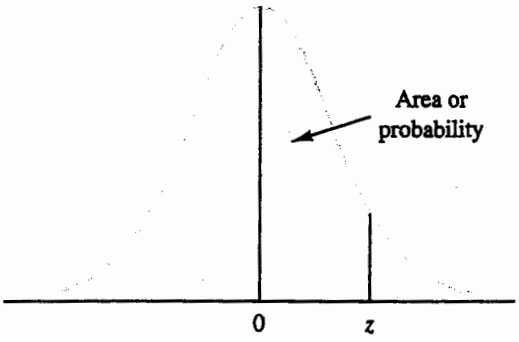

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean and z is .3944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990