UNIVERSITY OF SWAZILAND

FACULTY OF SOCIAL SCIENCE

DEPARTMENT OF ECONOMICS

SUPPLEMENTARY EXAMINATION PAPER: JULY, 2010

TITLE OF PAPER: QUANTITATIVE METHODS

COURSE CODE: ECON 205

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS:

1. Answer **FOUR** Questions. Two form Section A and two from Section B.

2. Show all relevant workings to your answer

3. All Questions carry a total of 25 marks

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATOR

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR.

SECTION A

Question 1

(a) Consider the following consumption function:

$$C = 100 + 0.8Y^d$$

i. What is the level of autonomous consumption? [2 marks]

ii. What is the value of the marginal propensity to consume? [2 marks]

iii. Compute the level of consumption expenditure when disposable income $Y^d = 100$ [3 marks]

iv. What will disposable income be if consumption expenditure is $C = 500$? [3 marks]

(b) Use Cramer's rule to solve for X and Y given the first order conditions for constrained optimization as follows:

$$\frac{\delta TC}{\delta X} = 16X - Y - \lambda = 0$$
$$\frac{\delta TC}{\delta Y} = 24Y - X - \lambda = 0$$
$$\frac{\delta TC}{\delta \lambda} = 42 - X - Y = 0$$

[15 marks]

Question 2

a) Solve the following equation $X^2 = 5X + 2\frac{3}{4}$ [9 marks]

b) A firms demand function is given by $Q_d = 120 - P$ and its total cost function is given by $TC = 2Q^2 + 6Q + 216$. If the firm produces what it can sell and no more.

i) determine the break-even point(s) for the firm [8 marks]

ii) determine the level of output where marginal revenue is at maximum [4 marks]

iii) determine the level of output where profit is maximized [4 marks]

Question 3

(a) Show that the following matrix is idempotent

$$\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$$

[5 marks]

(b)The input-output matrix for a two industry economy is expressed by the following table:

	INPUT TO		
	INDUSTRY 1	INDUSTRY 2	LEVEL OF OUTPUT
OUTPUT FROM			
INDUSTRY 1	300	800	2400
INDUSTRY 2	600	200	4000

i. Determine the level of final demand which can be met by the two industries. [5 marks]

ii. Determine the matrix of technology coefficients for the two industry economy. [5 marks]

iii. If the level of final demand for the output of the two industries is 5000 units for industry 1 and 400 units for industry 2, at what level of output should the two industries operate? [10 marks]

Question 4

a) A toy manufacturer in Matsapha makes two games: Bong and Zong. The profit margin on Bong is E30 per unit and the profit margin on Zong is E20 per unit. One Bong takes 6 hours of processing, 4 hours of assembling and 5 hours of packing. One Zong takes 3 hours of processing, 6 hours of assembling and 5 hours of packing. 54 hours are available for processing, 48 hours for assembling, and 50 hours for packing.

i. Formulate and solve the linear programming problem of maximizing profits subject to the given constraints [20 marks]

ii. Formulate the dual problem (do not solve) [5 marks]

SECTION B

Question 5

(a) Cholera , an intestinal disease, caused by a cholera bacterium that multiplies exponentially by cell division as given approximately by:

$N = N_0 e^{1.386t}$ where N is the number of bacteria present after t hours and N_0 is the number of bacteria present at the start ($t = 0$). If we start with 25 bacteria, determine the bacteria (to the nearest unit) that will be present in 0.6 hours and 3.5 hours respectively.

[10 marks]

(b) An amusement company maintains records for each video game it installs in an arcade. Suppose that C(t) and R(t) represent the total accumulated costs and revenues (in thousands of Emalangeni), respectively, t years after a particular game has been installed and that

$$C'(t) = 2 \qquad R'(t) = 9e^{-0.5t}$$

The value of t for which $C'(t) = R'(t)$ is called the useful life of the game, find ;

i. The useful life of the game to the nearest year [5 marks]

ii. The total profit accumulated during the useful life of the game [10 marks]

Question 6

(a) Find the optimizing values for minimizing the costs of a firm producing two goods x and y when the total cost function is $C = 8x^2 - xy + 12y^2$ and the firm is bound by contract to produce a minimum combination of goods totalling 42, that is, subject to the constraint $x + y = 42$. [15 marks]

(b) A manufacturer of PVRs purchases a particular microchip, called the LS-24 from three suppliers: X, Y and Z. 30% of LS-24 chips are purchased from X, 20% from Y and 50% from Z. Through experience, the manufacturer knows that 3% of the chips from X are defective, 5% of the chips from Y are defective, and 4% of the chips from Z are defective. If one of the PVR manufacturing workers randomly selects a chip for installation in a PVR and finds it defective, what is the probability that it was purchased from Y? [10 marks]

Question 7

The following table shows the weekly labour costs paid (X) and weekly revenues received from bicycle repairs (Y) by a bicycle shop for a quarter of a year.

LABOUR COSTS(E)	REVENUES(E)
X	Y
7.1	3.9
2.2	1.7
7.2	3.7
3.1	1.5
3.3	2.2
3.5	1.4
7.7	4.9
4.1	2.5
6.8	4.5
4.7	3.4
5.3	2.3
5.5	3.6
6.1	3.2

(a) Find the least-squares line which will enable us to predict repair revenues from labour costs of the bicycle shop. [15 marks]

(b) Interpret the results [3 marks]

(c) Find the coefficient of correlation and test its significance at 5% level [4marks]

(d) Calculate the coefficient of determination. [3 marks]

Question 8

(a) Write explanatory notes on the following

(i) Type one and type two error in hypothesis testing
(ii) Heteroscedasticity
(iii) Multicollinearity
(iv) Stochastic model
(v) Autocorrelation [3 marks each]

(b) Explain the significance of the error term (u) in econometric model specification and state the assumptions surrounding the error term in model specification. [10 marks]