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UNIVERSITY OF SWAZILAND

FACULTY OF SOCIAL SCIENCE

DEPARTMENT OF ECONOMICS

MAIN EXAMINATION

DECEMBER 2011

TITLE OF PAPER: MATHEMATICS FOR ECONOMISTS

COURSE CODE: ECON 208

TIME ALLOWED: THREE(3) HOURS

INSTRUCTIONS:

1. ANSWER THREE (3) QUESTIONS:

   QUESTION ONE(1) IS COMPULSORY AND YOU CAN THEN CHOOSE ANY TWO (2) QUESTIONS FROM THE REMAINING FOUR (4) QUESTIONS PROVIDED.

2. QUESTION 1 CARRIES 50 MARKS AND THE CHOSEN TWO QUESTIONS CARRY 25 MARKS EACH

3. IN EVERY STAGE OF YOUR CALCULATIONS ROUND YOUR ANSWER TO TWO (2) DECIMAL PLACES.

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QUESTION 1 (compulsory)

a) A Company has two inter-acting branches, B1 and B2. Branch B1 consumes E0.5 of its own output and E0.2 of B2 output for every E1 it produces. Branch B2 consumes E0.6 of B1 output and E0.4 of its own output per E1 of output. The company wants to know how much each branch should produce per month in order to meet exactly a monthly external demand of E50,000 for B1 product and E40,000 for B2 product.

i) Set up (without solving) a linear system whose solution will represent the required production schedule. **(5)**

ii) Calculate the amount of primary input required to produce the solution output levels. **(3)**

iii) Find a production schedule for the above external demand. **(15)**

b) i) Write short explanatory notes on the conformability condition for matrix multiplication. **(3)**

ii) Outline the properties of a transpose of a matrix **(6)**

iii) Evaluate the following determinant using the Laplace Expansion **(10)**

$$A = \begin{vmatrix} 15 & 7 & 9 \\ 2 & 5 & 6 \\ 9 & 0 & 12 \end{vmatrix}$$

c) Find the inverse of the following matrix **(8)**

$$A = \begin{bmatrix} 4 & 1 & -5 \\ -2 & 3 & 1 \\ 3 & -1 & 4 \end{bmatrix}$$

QUESTION 2

a) If the supply and demand equations for a product are :

$$Q_s = 50p - 40$$
$$Q_d = 480 - 15p$$

i) Determine the equilibrium values and producer's revenue at the equilibrium Point. **(7)**

ii) A Flat –rate tax of E4 per unit is imposed. Determine the new equilibrium values. **(4)**

iii) What effect does the flat rate tax have on the supplier's revenue and tax revenue **(4)**

b) In the following national income model, Y is national income, C is consumption, Y^d is disposable income, I is investment, G is government expenditure and T is tax.

$$Y = C + I + G$$
$$C = 10 + 0.6Y^d$$
$$I = 25$$
$$T = 2 + 0.2Y$$

i) If G = 25, determine the equilibrium level of national income **(3)**

ii) If G = T, determine the equilibrium level of national income. **(3)**

iii) Does changing the tax function to T = 2 + 0.15Y increase national income? **(4)**

QUESTION 3

a) Write short explanatory notes on the following: **(5 marks each)**

i) Parabola

ii) A compound interest

b) A Manufacturer has the following information about the costs of producing a machine:

Output (x)	5	10	15
Total Costs	20	65	160

i) Determine the equation of the cost function, assuming it can be represented by a quadratic expression. **(10)**

ii) What is the cost of producing 20 units of output? **(2)**

iii) If the revenue function is TR = 10x + 15
Show that there are two levels of output at which total revenue equals total costs **(8)**

QUESTION 4

a) Find the value of E100 at 10% interest for 2 years:

i) If compounded annually. **(2)**

ii) If compounded continuously **(2)**

b) The firm's demand function is given by $Q_d = 120 - P$ and its total cost function is

$TC = 2Q^2 + 6Q + 216$. If the firm produces what it can sell, and not more,

i) Determine the breakeven point(s) for the firm. **(10)**

ii) Determine the level of output where:

1) Marginal revenue is zero **(3)**
2) Average cost is at minimum **(3)**
3) Profit is maximized **(3)**

iii) What is the firm's profit when output is 25 units? **(2)**

QUESTION 5

A company manufactures two products X and Y. Each product has to be processed in three departments: welding, assembly and painting. Each unit of X spends two hours in the welding department, three hours in the assembly and one hour in the painting department. The corresponding times for a unit of Y are 3, 2 and 1 respectively. The man-hours available in a month are 1500 in the welding department, 1500 in the assembly and 550 in the painting department. The contribution to profits and fixed overheads are E100 for product X and E120 for product Y.

a) Formulate the linear programming problem and show it graphically by shading the feasible region. **(10)**

b) Find the optimal solution and the maximum contribution. **(10)**

c) Which department has spare capacity and how much? **(5)**