# UNIVERSITY OF SWAZILAND FACULTY OF SOCIAL SCIENCE DEPARTMENT OF ECONOMICS 

## MAIN EXAMINATION PAPER: December 2016

TITLE OF PAPER: MATHEMATICS FOR ECONOMISTS I COURSE CODE: ECO 205<br>TIME ALLOWED: TWO (2) HOURS<br>INSTRUCTIONS:<br>1. Answer question1 (COMPULSORY), and any other two questions<br>2. Show all relevant workings to your answer<br>3. Question 1 carries 50 marks and the rest of the questions are graded out of 25 marks

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATOR

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR

## Question 1

a) Write short explanatory notes ( not more than a quarter of a page each) on the following:
i) Deterministic and stochastic model
ii) Linear dependency and matrix singularity
iii) The Leontif model
iv) Static vs comparative static equilibrium
v) Identity and behavioural equation [5 marks each]
b) Given the following macroeconomic model:

$$
\begin{gathered}
Y=C+I+G \\
C=200+0.7 Y \\
I=75+0.1 Y \\
G=100
\end{gathered}
$$

Express the model in matrix form and hence find the equilibrium values of $C, I, G$, and $Y$.
c) It is known in the university bookshop that when the price of a certain book is E10 there will be no purchases, but it is thought that for every Lilangeni drop in price, twelve new purchases would appear. Furthermore it is known that the publishers refuse to offer any copies for sale at a price of E3 or less, but they are prepared to offer 9 copies for E4 and an additional nine copies for each one Lilangeni rise in price. Find two equations showing:
i) the quantity demanded $\left(Q_{d}\right)$ in terms of price.
ii) the quantity supplied $\left(Q_{s}\right)$ in terms of price.
[3 marks]
c) Find the equilibrium price at which quantity demanded will equal the quantity supplied.
[4 marks]

## ANSWER ANY TWO QUESTIONS FROM THE FOLLOWING:

## Question 2

A perfectly competitive firm has a demand function $P=121$ and cost given as
$T V C=1 / 2 Q^{3}-15 Q^{2}+175 Q, T F C=500$
a) Write the equations for $T C, T R$, and $\pi$
b) Find output, $Q$, at which profit is minimized and maximized, comment on the relationship between MC, and MR at these points.
[15 marks]
c) Show that $M C=M R$ is a necessary but not sufficient condition for maximum profit.
[5 marks]

## Question 3

a) Use Cramer's rule to solve for $X$ and $Y$ given the first order conditions for constrained optimization as follows:

$$
\begin{aligned}
& \delta T C / \delta X=16 x-y-\lambda=0 \\
& \delta T C / \delta Y=24 y-x-\lambda=0 \\
& \delta T C / \delta \lambda=42-x-y=0
\end{aligned}
$$

[8 marks]
b) A distributor records the weekly sales of personal computers (PCs) in three retail outlets in different parts of the country as shown in the table below:

Number of computers sold in each shop

|  | Basic | Extra | Latest |
| :--- | :---: | :---: | :---: |
| Shop A | 150 | 320 | 180 |
| Shop B | 170 | 420 | 190 |
| Shop C | 201 | 63 | 58 |

The cost price of each model is: Basic E480, Extra E600, Latest E1020. The retail price of each model in each of these three shops is given in the table below:

Selling price of computers in each shop

|  | Basic | Extra | Latest |
| :---: | :---: | :---: | :---: |
| Shop A | 560 | 750 | 1580 |
| Shop B | 520 | 690 | 1390 |
| Shop C | 590 | 720 | 1780 |

Use matrix multiplication to calculate
i) The total weekly cost of computers to each shop. [5 marks]
ii) The total weekly revenue for each model for each shop. [8 marks
iii) The total weekly profit for each shop. Which shop makes the greatest profit?

## Question 4

a) Solve the following equations by Gaussian elimination

$$
\begin{gathered}
2 x+y+z=12 \\
6 x+5 y-3 z=6 \\
4 x-y+3 z=5
\end{gathered}
$$

[10 marks]
b) Find the inverse of the matrix

$$
D=\left(\begin{array}{ccc}
1 & 0 & -2 \\
2 & 2 & 3 \\
1 & 3 & 2
\end{array}\right)
$$

By Gauss-Jordan elimination
c) Show that $D D^{-1}=I$

## QUESTION 5

The input-output matrix for a two industry economy is expressed by the following table:

| INDUSTRY 1 |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| INPUT TO |  |  |  |  |  |  |  | INDUSTRY 2 | LEVEL | OF |
| OUTPUT |  |  |  |  |  |  |  |  |  |  |
| OUTPUT FROM |  |  |  |  |  |  |  |  |  |  |
| INDUSTRY 1 | 300 | 800 | 2400 |  |  |  |  |  |  |  |
| INDUSTRY 2 | 600 | 200 | 4000 |  |  |  |  |  |  |  |

a) Determine the level of final demand which can be met by the two industries.
[5 marks]
b) Determine the matrix of technology coefficients for the two industry economy.
[5 marks]
c) If the level of final demand for the output of the two industries is 5000 units for industry 1 and 400 units for industry 2, at what level of output should the two industries operate?
[15 marks]

