UNIVERSITY OF SWAZILAND
FACULTY OF SOCIAL SCIENCES
DEPARTMENT OF ECONOMICS
MAIN EXAMINATION 2016/2017

TITLE OF PAPER : MATHEMATICS FOR ECONOMISTS II

COURSE CODE : ECO 206

TIME ALLOWED : TWO (2) HOURS

INSTRUCTIONS :

1. **ANSWER ANY FOUR (4) QUESTIONS IN THIS PAPER. QUESTIONS CARRY 25 MARKS EACH.**
2. **ONLY SCIENTIFIC NON-PROGRAMMABLE CALCULATORS ARE ALLOWED.**
3. **ROUND UP YOUR FINAL ANSWERS TO TWO (2) DECIMAL PLACES.**
4. **IF IT IS NOT SPECIFIED, USE $\alpha = 0.05$ FOR STATISTICAL TESTS.**
5. **THE REQUIRED PROBABILITY TABLES ARE ATTACHED AT THE BACK OF QUESTION PAPER.**

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

QUESTION 1 [25 MARKS]

An analyst monitored 10 sewing machine operators at a certain garment factory in Matsapha to determine how many shirts per day, each worker produced. The results are recorded as follows:

175 190 250 230 240 200 185 190 225 265

a) Find the average number of shirts produced a day? [4 Marks]

b) What is the median number of shirts produced? [5 Marks]

c) If you were required to produce a report describing the data on this experiment, which measure of central tendency would you use? Explain why. [4 Marks]

d) What proportion of the machine operators lie within two (2) standard deviations of the mean number of shirts produced? [8 Marks]

e) Does this proportion you obtained in (d) above agree with the proportions given by Tchebysheff's theorem? [4 Marks]

QUESTION 2 [25 MARKS]

a) Distinguish between mutually exclusive events and independent events. [6 Marks]

b) Define Conditional Probability. [3 Marks]

c) In a certain undergraduate economics class, 15% of the students are considered to be at a high risk of re-sitting the course, based on their test scores. Three (3) students are selected at random from this class, what is the probability that **exactly two (2)** of the three students chosen are at high risk of re-sitting the course? [8 Marks]

d) If additionally we know that 51% of all the students are female, and that 12% of the females are at a high risk of a re-sit. If a student is selected at random, what is the probability that they are a female who is considered to be at a high risk for a re-sit?

[8 Marks]

QUESTION 3 [25 MARKS]

A street vendor in the Manzini Market packs tomatoes in small plastic bags. However, since the tomatoes are not equal in sizes, the weight of each packed plastic bag varies. The weights of the packed tomatoes is normally distributed with a mean of $1Kg$ and a standard deviation of $0.15Kg$.

a) What proportion of the packets will weigh more than $1Kg$? [7 Marks]

b) What proportion of the packets will weigh between $0.95Kg$ and $1.05Kg$? [6 Marks]

c) What is the probability that a randomly selected packet of tomatoes will weigh less than $0.80Kg$? [6 Marks]

d) If you were to select a packet at random and you found that it weighs $1.45Kg$, would this be a usual or unusual occurrence? Justify your answer. [6 Marks]

QUESTION 4 [25 MARKS]

a) Describe what is a *p*-value. [5 Marks]

b) In a survey conducted by the Ministry of Health, 9% of parents describe their children as being overweight. However, results from another study conducted by researchers at the Faculty of Consumer Sciences in Luyengo claim that obesity levels in children are at least 15%. Suppose that you randomly sample 750 parents ($n = 750$), and observe that 68 of the parents describe their children as overweight.

   i. Formulate and test the hypothesis that the proportion of parents who describe their children as overweight is less than the actual proportion reported by the Luyengo researchers. [15 Marks]

   ii. Use the *p*-value from the test to come up with a conclusion. [5 Marks]

QUESTION 5 [25 MARKS]

a) List the five (5) components involved in a statistical test. [5 Marks]

b) Distinguish between Type I and Type II Errors in a statistical test. [6 Marks]

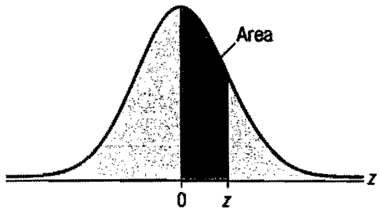
c) Two diet programs designed for individuals aged between 20 – 30 years are compared. The following sample data were for the two (2) diets were obtained

	Sample Size (n)	Sample Mean($\bar{x}$)	Sample Variance(s^2)
Diet 1	40	10	4.3
Diet 2	40	8	5.7

Do the data provide sufficient evidence to suggest that Diet 1 produces a greater mean weight loss than Diet 2? [14 Marks]

C Standard Normal Distribution

Numerical entries represent the probability that a standard normal random variable is between 0 and z where $z = \frac{x-\mu}{\sigma}$.

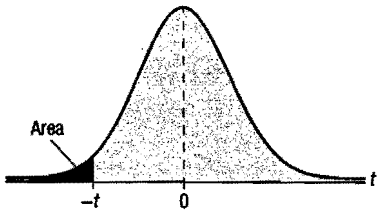


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

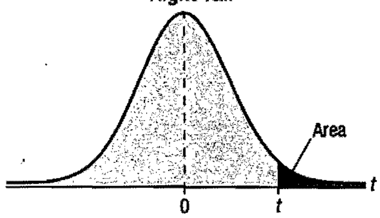
D Critical Values of *t*

	Area in One Tail				
	0.100	0.050	0.025	0.010	0.005
	Area in Two Tails				
df	0.200	0.100	0.050	0.020	0.010
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.434	2.719
38	1.304	1.686	2.024	2.429	2.712
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.368	2.832
100	1.290	1.660	1.984	2.364	2.626
120	1.289	1.658	1.980	2.358	2.617
200	1.286	1.653	1.972	2.345	2.601
300	1.284	1.650	1.968	2.339	2.592
400	1.284	1.649	1.966	2.336	2.588
500	1.283	1.648	1.965	2.334	2.586
750	1.283	1.647	1.963	2.331	2.582
1000	1.282	1.648	1.962	2.330	2.581
∞	1.282	1.845	1.960	2.326	2.576

Left Tail



Right Tail



Two Tails