## UNIVERSITY OF SWAZILAND

## FACULTY OF SOCIAL SCIENCES

## DEPARTMENT OF ECONOMICS

SUPPLEMENTARY EXAMINATION 2016/2017

| TITLE OF PAPER | $:$ | MATHEMATICS FOR ECONOMISTS II |
| :--- | :--- | :--- |
| COURSE CODE | $:$ | ECO 206 |
| TIME ALLOWED | $:$ | TWO (2) HOURS |

INSTRUCTIONS :

1. ANSWER ANY FOUR (4). QUESTIONS IN THIS PAPER. QUESTIONS CARRY 25 MARKS EACH.
2. ONLY SCIENTIFIC NON-PROGRAMMABLE CALCULATORS ARE ALLOWED.
3. ROUND UP YOUR FINAL ANSWERS TO TWO (2) DECIMAL PLACES.
4. IF IT IS NOT SPECIFIED, USE $\alpha=0.05$ FOR STATISTICAL TESTS.
5. THE REQUIRED PROBABILITY TABLES ARE ATTACHED AT THE BACK OF QUESTION PAPER.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

## QUESTION 1

a) Distinguish between mutually exclusive events and independent events.
b) Define Conditional Probability.
c) In a certain citrus canning factory in Malkerns, $8 \%$ of the product is considered defective, based on samples taken by the company laboratory. If three (3) cans are selected at random, what is the probability that exactly one (1) of the cans chosen is defective?
[8 Marks]
d) If additionally we know that $40 \%$ of the canned products pass through machine 2 , and that $12 \%$ of the cans through this machine are defective. If a can is selected at random, what is the probability that it was closed by machine 2 and it is defective? [8 Marks]

## QUESTION 2

[25 MARKS]

Records from The Clinic Group of hospitals show that 30\% of all patients admitted in their hospitals fail to settle their bill, and eventually these bills have to be forgiven. Suppose four (4) new patients $(n=4)$ represent a random selection from the large set of prospective patients served by the hospitals, find the probabilities:
a) All the patient's bills will have to be forgiven.
b) One (1) will have to be forgiven.
c) None will have to be forgiven.
d) Suppose that over a one year period, the hospitals admit 2000 patients,
i. What is the mean number of debts that have to be forgiven?
ii. What is the variance and standard deviation of forgiven debts?

## QUESTION 3

A price analyst for a certain marketing firm in Mbabane samples the price of different brands of $100 \% 1$ Litre fruit juices at various large supermarkets. The figures obtained in Emalangeni, are as follows:
9.90
$19.20 \quad 12.30 \quad 8.50$
6.00
$6.50 \quad 5.30$
$14.10 \quad 11.20 \quad 6.30$
6.70
6.90
6.00
6.00
6.60
a) Find the average price for the different brands of fruit juices?
b) What is the median price for the different brands of fruit juices?
c) If you were required to produce a report describing the data on this experiment, which measure of central tendency would you use? Explain why.
[4 Marks]
d) What proportion of the fruit juice prices lie within two (2) standard deviations of the mean price?
e) Does this proportion you obtained in (d) above agree with the proportions given by Tchebysheff's theorem?
[4 Marks]

## QUESTION 4

[25 MARKS]
a) Distinguish between Type I and Type II Errors in a statistical test.
b) List the five (5) components involved in a statistical test.
c) Independent random samples of $n_{1}=50$ and $n_{2}=60$ observations were selected from populations 1 and 2, respectively. The sample data were for the two (2) populations are computed and given below:

|  | Sample Size (n) | Sample Mean $(\overline{\boldsymbol{x}})$ | Sample Variance $\left(\boldsymbol{s}^{\mathbf{2}}\right)$ |
| :---: | :---: | :---: | :---: |
| Population 1 | 50 | 100.4 | 0.8 |
| Population 2 | 60 | 96.2 | 1.3 |

Find the $95 \%$ confidence interval for the difference in population means and interpret the interval.
[14 Marks]

## QUESTION 5

[25 MARKS]
a) Describe what is a $p$-value.
b) The Government of Swaziland is in the process of resuscitating the national airline, Swazi Airways. Suppose that for the Sikhuphe - Johannesburg route to be viable, flights need at least $60 \%$ occupancy rate. If a sample of 120 flights in this route is taken and it yields an average flight occupancy of $58 \%$ with a standard deviation of $11 \%$. Using a $5 \%$ significance level, determine whether the Sikhuphe - Johannesburg flight is viable?
[15 Marks]
c) Use the $p$-value from the test in (b) above and see whether you reach the same conclusion.
[5 Marks]

## Standard Normal Distribution

Numerical entries represent the probability that a standard normal random variable is between 0 and $z$ where $z=\frac{x-\mu}{\sigma}$.


| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 000 | dope0 | 010040 | 0.9080 | 0.120 | 0.0180 | 00199 | 00239 | 0.0279 | 0.03 | 0.9359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.055 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 002 | 3 |  | - | 009 | 808 | 0.0 | 0 | 207164 | 0.1 | 01141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 5049 | 0.1554 | 0159 | 016828. | 0.166 | 0.17 | 0.1 | 0.172 | 0188 | 0.1844 | 01879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| \% 8 | ,602257 | 02291 | 0.2324 | 0.2357 | 023889 | 0.2422 | 0.2454 | 0.2486 | $0: 2517$ | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 408 | W0.2831 | 02980 | 2,2999 | 0.296 | 02295 | 0.3023 | $0,365{ }^{\circ}$ | 03078 | 0.3106 | 0.3138 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 10 | 03343 | 0.3438 | 0346 | 0.3465 | 0,3508 | 03531 | 0.3554 | 0.357 | 03599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 4, 12 | Q0a849 | 603869 | 03688 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 03997 | 04015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 147 | 04192 | 04207 | 04222 | 0423 | 04251 | 0.4265 | 04279 | 0.429 | 04300 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| W16 | - 0.4452 | * | 04474 | 04 | 044485 | 0 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 118 | - ${ }^{\text {aka }}$ | 0.9649 | 6.4856 | 0,4664 | 0.4671 | 0.4678 | , | 0.4693 | 04699 | 04706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 20 | - 0.4772 | 04778 | 0.4783 | 0.4788 | 0.4793 | 0.4788 | 04803 | 0.4808 | 0.4812 | 04817 |
| 21 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 22. | - 04861 | -04864 | 04868 | 0.487 | 04875 | 04878 | 0.4887 | 0.48 | 0.4887 | 0.4890 |
| 23 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| , 24 | 0.4018 | 0,4920 | 04922 | 0.4925 | 0.4927 | 04929 | 0.4931 | 0.4932 | 0.4934 | $0: 4936$ |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 3260 |  |  |  | 0,485 | , | 4460 |  | 049 | 0.4483 | 4.964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 288\% | 609974 | 1803975 | 049 | 04977 | 0.4977 | 0.4978 | 04979 | 0.4979 | 0.4980 | 04981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| W90\% | Wbutab7 | 0:4987 | $0.4$ | + 0 | $0.49$ | 0 | \% 048 | 049 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 822 | Wh4993. | 0.4993 | 04994 | +0.4994 | 204994 | - 44994 | 0.4994 | 0.4995 | 0.4995 | 04995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4998 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 34 | W, 0.4997 | 0.4997 | 04497 | 04997 | 04997 | 04997 | 0.49 | 0.4997 | 0.4997 | 0.4998 |

## D Critical Values of $t$



