

UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2005

B.A.S.S. I / D.COM I

TITLE OF PAPER : CALCULUS FOR BUSINESS AND SOCIAL SCIENCE

COURSE NUMBER : MS 102 AND IDE MS100-2

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS
3. SHOW ALL THE RELEVANT WORKING

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Evaluate the following limits:

(i) $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 4x + 3}$ [4 marks]

(ii) $\lim_{x \rightarrow \infty} \frac{1 - 2x^2 + 3x^3 - 4x^4}{2x^4 + x^2 - x + 1}$ [4 marks]

(b) Use the **limit definition** of the derivative to find the derivative $f'(x)$ corresponding to the following functions.

(i) $f(x) = \sqrt{x+1}$ [6 marks]

(ii) $f(x) = \frac{x}{x+1}$ [6 marks]

QUESTION 2

2. (a) Find the derivatives of the following functions

(i) $y = x^3 e^{x^3}$ [5 marks]

(ii) $y = x^{x^2}$ [5 marks]

(iii) $y = \sin^5(x^2 + e^{x^2} + \ln x + 1)$ [4 marks]

(b) Find the first three (3) derivatives of $y = (3x + 2)^{10}$ [6 marks]

QUESTION 3

3. Find the following integrals

(a) $\int \left(3x^2 + e^{2x} + \sin 2x + \frac{3}{x} + \frac{1}{x^2} \right) dx$ [5 marks]

(b) $\int (x^2 + x + 1) e^x dx$ [5 marks]

(c) $\int \frac{x + 4}{x^2 + 5x - 6} dx$ [5 marks]

(d) $\int \frac{2x + 1}{\sqrt{x^2 + x + 1}} dx$ [5 marks]

QUESTION 4

4. (a) The marginal cost of producing x items of a product is given by $C'(x) = 0.04x$.

(i) Given that the fixed cost is E 20, find the total cost function $C(x)$. [5 marks]

(ii) Find the cost of producing 100 of these items. [2 marks]

(iii) Find the total change in cost if the number of items produced is changed from 100 to 200. [3 marks]

(b) A company manufactures and sells x computers per week. If the weekly cost and price-demand functions are given by

$$C(x) = 5000 + 2x \quad \text{and} \quad p = 10 - 0.001x$$

Find the following, for each week:

(i) the maximum revenue [4 marks]

(ii) the maximum profit [4 marks]

(iii) the price that will yield maximum profit [2 marks]

QUESTION 5

5. (a) Given the function $f(x) = x^3 - 3x + 2$, find
- (i) the y -intercept [1 mark]
 - (ii) stationary points [2 marks]
 - (iii) intervals of increase and decrease [2 marks]
 - (iv) relative extrema [2 marks]
 - (v) inflection points [1 mark]
- (b) Use all the information obtained in (a) to sketch a graph of the function. [4 marks]
- (c) A moving company wishes to design an open-top box with a square base whose volume is exactly 32 cubic metres. Find the dimensions of the box requiring the least amount of materials. [8 marks]

QUESTION 6

6. (a) Find the area of the region bounded by the parabola $y = x^2$ and the line $y = x + 2$ [5 marks]
- (b) A company manufactures x printers per month. The marginal monthly profit (in Emalangen) is given by

$$P'(x) = 165 - 0.1x$$

- The company is manufacturing 1500 printers per month, but is planning to increase production. Find the total change in monthly profit if monthly production is increased to 1600 printers. [5 marks]
- (c) Find the consumer's surplus at a price level of $p^* = 20$ for the demand equation $p = D(x) = 40 - 4x$ [5 marks]
- (d) Find the producer's surplus at a price level of E 20 for the supply equation $p = S(x) = 2 + 0.0002x^2$ [5 marks]

QUESTION 7

7. Find the following integrals

(a) $\int \sin^5 x \, dx$ [5 marks]

(b) $\int_1^2 (x^2 + 2x - 1) \, dx$ [5 marks]

(c) $\int (2x + 1)e^{x^2+x+1} \, dx$ [5 marks]

(d) $\int x^2 \sin x \, dx$ [5 marks]