

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2005

B.A.S.S. I / D.COM I

TITLE OF PAPER : CALCULUS FOR BUSINESS AND SOCIAL SCIENCE

COURSE NUMBER : MS 102 AND IDE MS100-2

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : 1. THIS PAPER CONSISTS OF
SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Evaluate the following limits:

(i) $\lim_{x \rightarrow -2} \frac{x^2 - 3x - 10}{x + 2}$ [5 marks]

(ii) $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$ [5 marks]

(b) Use the limit definition of the derivative to find $f'(x)$ where

(i) $f(x) = x^2 + 1$ [5 marks]

(ii) $f(x) = \sqrt{x}$ [5 marks]

QUESTION 2

2. Find the derivatives, $f'(x)$, of the following functions

(a) $f(x) = x^2 \cos 3x$ [5 marks]

(b) $f(x) = \frac{e^{2x}}{1 + e^{x^2}}$ [5 marks]

(c) $f(x) = \ln \frac{(x^2 + 1)^5}{\sqrt{x^2 - 2}}$ [5 marks]

(d) $f(x) = x^3 \ln x^3$ [5 marks]

QUESTION 3

3. (a) A company manufactures and sells x radios per week. If the weekly cost and price-demand functions are given by

$$C(x) = 5000 + 2x, \quad p = 10 - 0.001x$$

Find the following, for each week.

- (i) the revenue function. [2 marks]
 - (ii) the maximum revenue. [4 marks]
 - (iii) the maximum profit. [4 marks]
 - (iv) the price that will yield maximum profit. [2 marks]
- (b) Sketch the graph of $y = x^3 - 6x^2 + 9x + 1$, showing all relative extreme points, inflection points and the y -intercept. [8 marks]

QUESTION 4

4. (a) Find the first four (4) derivatives of the function $y = (2x + 1)^5$. [8 marks]
- (b) From a thin piece of cardboard that is 12cm by 12cm, square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? [12 marks]

QUESTION 5

5. Evaluate the following integrals

- (a) $\int \left(2 - x^2 + \frac{1}{x} + \frac{4}{x^4} \right) dx$ [5 marks]
- (b) $\int xe^x dx$ [5 marks]
- (c) $\int \left(\frac{2x - 1}{x^2 - 3x + 2} \right) dx$ [5 marks]
- (d) $\int x(x^2 + 1)^4 dx$ [5 marks]

QUESTION 6

6. (a) Find the area of the region bounded by the parabola $y = 9 - x^2$ and the x -axis [8 marks]

(b) Find the equation of the tangent to the curve $y = x^2 + 5x + 5$ at $x = -1$ [6 marks]

(c) If the marginal cost of producing x units is given by

$$C'(x) = 0.3x^2 + 2x + 1$$

and the fixed cost is E1000, find the cost function $C(x)$ [6 marks]

QUESTION 7

7. Given the demand function $D(x) = 20 - 0.05x$ and the supply function $S(x) = 2 + 0.0002x^2$, find

(a) the equilibrium price. [6 marks]

(b) the consumer's surplus [7 marks]

(c) the producer's surplus [7 marks]