THE UNIVERSITY OF SWAZILAND 4^{b}

Department of Mathematics

Final Examination 2005

MS202
QUANTITATIVE TECHNIQUES

Three (3) hours

INSTRUCTIONS
1. This paper contains SEVEN questions in TWO sections.
2. Answer FIVE questions, at least TWO from each section.

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

Section A

Answer at least TWO and no more than THREE questions from this section.

Question 1. (a) The demand functions for two products are

$$p = 12 - x, \qquad q = 34 - y$$

where p and q are the respective prices (in thousands of Emalangeni) and x and y are the respective amounts (in thousands of units) of each product sold. If the joint cost function is given by

$$C(x,y) = x^2 + 2xy + 3y^2,$$

determine the quantities and prices that maximize profit (Verify they give maximum profit). What is the maximum profit?

[12 marks]

(b) A debt of E20,000 at 15% p.a. (simple) is due in 18 months. The debtor pays E5,000 at the end of 6 months, and another E8,000 at the end of one year. Find the balance at due date, assuming payments earn interest.

[8 marks]

Question 2. (a) Suppose the 4th and 6th terms in an arithmetic progression are 4 and 4.8 respectively. Find the first term.

[5 marks]

(b) The management of a factory wants to set up a fund to provide E40,000 for the replenishment of a machine at the end of five years. If equal deposits are made at the end of each period of SIX months in a fund earning 4% converted semi-annually, find the size of each deposit.

[5 marks]

(c) A man can buy a plot of land for E40,000 cash or for E42,000 in a year's time. Assuming he has the cash which he has the option of investing at 6%, which option is more advantageous, and by how much?

[5 marks]

(d) Maximize $z = 4x^2 - 2xy + 6y^2$ subject to $x + y = 72$ using the method of Lagrange multipliers.

[5 marks]

Question 3. (a) Use Gaussian elimination to solve

$$2x - z = 1$$
$$2x + 4y - z = 1$$
$$x - 8y - 3z = -2$$

[10 marks]

(b) For the function, $z = 3x^3 - 5y^2 - 225x + 70y + 23$, determine all local extrema and classify them accordingly.

[10 marks]

Question 4. (a) Let D, S and P denote demand, supply and price of each commodity in a two-commodity market (commodity 1 and 2) with linear model

$$D_1 = 20 - 2P_1 - P_2$$
$$S_1 = 4P_1 - P_2 + 2$$
$$D_2 = 8 + 5P_1 - 2P_2$$
$$S_2 = 3P_2 - 2$$

Determine the equilibrium solution.

[10 marks]

(b) Consider a two-industry economy, the information relating to the flow of goods for a particular output level is presented in the table below:

	Industry 1	*Industry 2*	*Output Level*
Industry 1	200	400	1,600
Industry 2	600	100	2,700

(i) Find the number of units exported to outside consumers, and hence complete the table.

(ii) Obtain the technology matrix for this economy.

(iii) Find the levels of output which would be required in order for Industry 1 to export 2000 units and Industry 2 to export 1000 units to consumers.

[10 marks]

Section B

Answer at least TWO Questions from this section.

Question 5. Use the simplex method to maximize $z = 5x_1 + x_2 + 8x_3$

subject to the following constraints

(a) $3x_1 - 3x_2 + 3x_3 \leq 220$

(b) $4x_1 + 3x_2 - 2x_3 \leq 200$

(c) $12x_1 - x_2 + 6x_3 \leq 40$

(d) $x_1, x_2, x_3 \geq 0$

[20 marks]

Question 6. At a department store, a three-category breakdown of the 10000 accounts receivable on 1st April showed that 80% payed on time, 15% were delinquent, and 5% were considered bad debts. On 1st May 6400 of those who payed on time were still paying on time, 1200 had become delinquent, and the remaining 400 were now considered bad debts. Of those who were delinquent on 1st April, 600 had started to pay on time, 600 remained delinquent, and 300 became bad debts. Of the bad debts, 100 had started to pay on time and the other 400 remained bad debts.

(a) Determine how many customers were in each category on 1st April, and construct the transition probability matrix.

[8 marks]

(b) How many customers were in each category on 1st May ?

[4 marks]

(c) If the system goes on operating under the same conditions for a large number of months, how many customers will be in each of the three categories?

[8 marks]

Question 7. (a) A company that manufactures jackets requires 50,000 zippers a month. The ordering cost is E5000 and inventory holding costs are estimated to be 5^C per unit per month. Find the most economical order quantity associated with this business.

[5 marks]

(b) Tambuti Citrus Corporation used 240,000 bags of fertilizer each year for its orange growers. It costs the company E100 to place an order and $E12^C$ to store 100 bags for a year. How often should the company place an order if it wishes to minimize the costs?

[5 marks]

(c) Imphilo Clinic employs one clerk, who handles all appointments for several doctors in the clinic. Incoming calls are answered by the switchboard operator. If a caller wants a doctor's appointment, the call is transferred to the clerk. If the clerk's line is busy, the operator asks the caller to wait. The operator transfers the waiting calls to the clerk on a first-come-first-serve basis when the clerk's line becomes free.

The calls for the appointments come in at a rate of nine per hour, and the average length of time it takes for the clerk to arrange an appointment is five minutes. Determine

(i) The probability that a person calling for an appointment will have to wait before reaching the clerk.

(ii) The average number of callers waiting to have their calls transferred to the clerk.

(iii) The average length of time a caller must be on the phone to receive an appointment.

[10 marks]

Some useful formulae.

Hire Purchase Instalment: $p = \dfrac{B\left(1 + \frac{n}{m}i\right)}{n + \frac{n(n-1)}{2m}i}$

Growing Investment: $S = \left(P + \dfrac{p}{i}\right)(1+i)^n - \dfrac{p}{i}$

Future Value of an Annuity: $S = p \cdot \dfrac{(1+i)^n - 1}{i}$

Present Value of an Annuity: $P = p \cdot \dfrac{1 - (1+i)^{-n}}{i}$

For an EOQ model without shortages: $\quad Q_0 = \sqrt{\dfrac{2dK}{h}}$

For a queueing model with one service channel and arrival and service rates λ, μ respectively:

$$\rho = \frac{\lambda}{\mu}; \qquad p_0 = 1 - \frac{\lambda}{\mu} \quad \text{and} \qquad p_n = \left(\frac{\lambda}{\mu}\right)^n p_0$$

$$L_s = \frac{\lambda}{\mu - \lambda} \qquad L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \qquad W_s = \frac{L_s}{\lambda} \qquad W_q = \frac{L_q}{\lambda}$$

(END)