THE UNIVERSITY OF SWAZILAND

Department of Mathematics

Supplementary Examination 2005

MS202
QUANTITATIVE TECHNIQUES

Three (3) hours

INSTRUCTIONS
1. This paper contains SEVEN questions in TWO sections.
2. Answer FIVE questions, at least TWO from each section.

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

Section A

Answer at least TWO and no more than THREE questions from this section.

Question 1. (a) The demand functions for two products are

$$p = 20 - x, \qquad q = 19 - y$$

where p and q are the respective prices (in thousands of Emalangeni) and x and y are the respective amounts (in thousands of units) of each product sold. If the joint cost function is given by

$$C(x, y) = 2x^2 + xy + 3y^2 + 50,$$

determine the quantities and prices that maximize profit (Verify they give maximum profit). What is the maximum profit?

[12 marks]

(b) A hifi, priced at E4000, is purchased on hire purchase with a deposit of E500. If equal payments are to be made over 24 months and interest is charged at 18%, calculate the monthly installment.

[8 marks]

Question 2. (a) Find the sum of the first 20 terms in an arithmetic progression in which the 5th term is 17 and the 9th is 29.

[6 marks]

(b) A company takes out a loan of E300,000 to replace a major plant, due in 10 years' time. The financier charges interest at 8%, compounded semiannually. Meanwhile, the company makes semiannual payments to a sinking fund that pays 10% interest, also compounded semiannually. Compute the total semiannual cost of the debt (Interest + Sinking fund).

[8 marks]

(c) Maximize $z = 4x^2 - 3x + 5xy - 8y + 2y^2$ subject to $x = 2y$ using the method of Lagrange multipliers.

[6 marks]

CONT ...

Question 3. (a) Use Gaussian elimination to solve

$$\begin{aligned} x + 2y + 3z &= 2 \\ 2x + 5y + 7z &= 1 \\ 2x + 4y + 5z &= 1 \end{aligned}$$

[10 marks]

(b) For the function, $z = 5x - x^2 + 3xy - 50y - y^2$, determine all local extrema and classify them accordingly.

[10 marks]

Question 4. (a) Evaluate the determinant of the matrix $A = \begin{bmatrix} 4 & -3 & 9 \\ 1 & -5 & -7 \\ 2 & 1 & 5 \end{bmatrix}$, by first transforming A to triangular form.

[10 marks]

(b) Consider a three-industry economy, the information relating to the flow of goods for a particular output level is presented in the table below:

	Industry A	*Industry B*	*Industry C*	*Output Level*
Industry A	100	150	70	500
Industry B	400	100	280	1,000
Industry C	100	300	140	700

(i) Find the number of units exported to outside consumers, and hence complete the table.

(ii) Obtain the technology matrix for this economy.

(iii) Find the consumer export levels that correspond to total outputs of 800 units for Industry A, 900 units for Industry B and 800 units for Industry C.

[10 marks]

CONT ...

Section B

Answer at least TWO Questions from this section.

Question 5. Use the simplex method to maximize $z = 5x_1 + 2x_2 + 8x_3$

subject to the following constraints

(a) $3x_1 - 6x_2 + 3x_3 \leq 220$

(b) $2x_1 + 3x_2 - x_3 \leq 100$

(c) $6x_1 - x_2 + 3x_3 \leq 20$

(d) $x_1, x_2, x_3 \geq 0$

[20 marks]

Question 6. Subscribers to digital TV have a choice three levels of service: standard, deluxe or super-deluxe. Of 10000 subscribers in the year 2004, 20% opted for the standard service, 50% the deluxe service and 30% subscribed to super-deluxe. At renewal for the following year, 80% of super-deluxe subscribers stayed at this level, while 20% downgraded to deluxe. Of those who subscribed to the deluxe service, 15% upgraded to super-deluxe, 10% downgraded to standard, and the rest stayed at deluxe level. At renewal, 30% of the standard subscribers upgraded to the deluxe service, and the rest stayed at the standard level.

(a) Determine how many customers subscribed to each level of service for the year 2004.

[3 marks]

(b) How many customers subscribed to each level of service for the year 2005.

[6 marks]

(c) Construct the probability matrix giving the change in subscribers for each level of service.

[3 marks]

(d) Assuming that pattern of changes of level of subscription at renewal continues each year, how many subscribers will there be at each level of service after a large number of years? [8 marks]

CONT ...

Question 7. (a) A university print room uses 50 boxes of paper each week, with each box costing E75. The cost of ordering a new supply of paper is E100 and the cost of storing 100 boxes of paper is E16 per week. Find the most economical order quantity for the print room.

[5 marks]

(b) The Save-Way supermarket sells 600 sacks of potatoes each month. The cost of ordering a fresh supply is E150 and the monthly cost of storing a sack of potatoes is E2. How many times per month should Save-Way place an order of potatoes if it wishes to minimize the costs?

[5 marks]

(c) The CalGen petrol station has one pump. Customers arrive on average at the rate of eight per hour and on average it takes six minutes to serve each customer. If someone is being served at the pump when a customer arrives, he waits in a queue.

Determine

(i) The probability that a customer will not have to wait before being served.

(ii) The average number of customers at the CalGen station (being served and waiting).

(iii) The average length of time a customer will have to queue before being served.

[10 marks]

CONT ...

Some useful formulae.

Hire Purchase Instalment: $p = \dfrac{B\left(1 + \frac{n}{m}i\right)}{n + \frac{n(n-1)}{2m}i}$

Growing Investment: $S = \left(P + \dfrac{p}{i}\right)(1+i)^n - \dfrac{p}{i}$

Future Value of an Annuity: $S = p \cdot \dfrac{(1+i)^n - 1}{i}$

Present Value of an Annuity: $P = p \cdot \dfrac{1 - (1+i)^{-n}}{i}$

For an EOQ model without shortages: $Q_0 = \sqrt{\dfrac{2dK}{h}}$

For a queueing model with one service channel and arrival and service rates λ, μ respectively:

$$\rho = \frac{\lambda}{\mu}; \qquad p_0 = 1 - \frac{\lambda}{\mu} \text{ and } \qquad p_n = \left(\frac{\lambda}{\mu}\right)^n p_0$$

$$L_s = \frac{\lambda}{\mu - \lambda} \qquad L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \qquad W_s = \frac{L_s}{\lambda} \qquad W_q = \frac{L_q}{\lambda}$$

(END)