UNIVERSITY OF SWAZILAND

Final Examination 2006

Title of Paper	:	Elementary Quantitative Techniques
Program	:	B.A.Hums./B.A.S.S. II
Course Number	:	M S001(i)
Time Allowed	:	Three (3) Hours
Instructions	:	

1. This paper consists of seven (7) questions on THREE (3) pages.
2. Answer ANY FIVE questions.
3. Non-programmable calculators may be used.

Special Requirements : None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

Question 1

(a) Express the following as single fractions in their simplest forms:

(i) $\dfrac{x}{x+1} - \dfrac{3}{x+2}$

[4 marks]

(ii) $1 - \dfrac{\sin^2 \theta}{\cos^2 \theta}$

[5 marks]

(b) Simplify the fraction $\dfrac{x^{-3}}{y^{-2}}$ by writing it as a positive index.   [5 marks]

(c) Simplify:

$$\frac{x^5 y^4 r^6}{4d^5 p^3} \div \frac{4p^2 r^7}{3y^2 x^5}$$

[6 marks]

Question 2

(a) Find the gradient and y-intercept of the line $2x + 3y = 5$.

[5 marks]

(b) Find the equation of a straight line that is

(i)   parallel to the line $2x + 3y = 5$ and passing through the point (4, 6).

[7 marks]

(ii)   perpendicular to the line $2x + 3y = 5$ and passing through the point (4, 6).

[8 marks]

Question 3

(a) Factorize the following expressions

(i) $3x^3 y^5 r^8 - 6x^6 y^2 qr^4$

[3 marks]

(ii) $\cos^2 x - \sin^2 x$

[3 marks]

(iii) $3k^2 x - 15k^2 y - x + 5y$

[4 marks]

(b) Given that $f(x) = \dfrac{x-3}{x+2}$, find

(i) $f(0)$ [2 marks]

(ii) $f^{-1}(x)$ [5 marks]

(iii) $f^{-1}(2)$ [3 marks]

Question 4

(a) Use long division to find the quotient and remainder when

$$3x^3 + 2x^2 + x + 4 \text{ is divided by } x - 2.$$

[10 marks]

(b) Use the remainder theorem to find the remainder when $4x^3 + 2x^2 - x - 4$ is divided by

   (i) $x + 3$ **[5 marks]**

   (ii) x **[5 marks]**

Question 5

(a) Solve the equation

$$2\cos x = 1 \text{ for } 0 \le x \le 360°.$$

[5 marks]

(b) Prove the following identities

   (i) $\cos^2 x - \sin^2 x \equiv 1 - 2\sin^2 x$ **[6 marks]**

   (ii) $\dfrac{1 - \tan^2 x}{\cot^2 x - 1} \equiv \tan^2 x$ **[9 marks]**

Question 6

(a) A student gets an allowance of $ $\dfrac{24}{x}$. He loses 30 cents; show that he remains with

$ $\dfrac{240 - 3x}{10x}$.   **[4 marks]**

(b) A student invests E 400 into an account that offers 7 % simple interest. How much will the student get if he withdraws everything after 8 months?

[6 marks]

(c) After how many years will a principal amount of E 8,500 double if invested into an account that offers 12 % interest compounded semi-annually?

[10 marks]

Question 7

(a) Correct the following statements (equations) where necessary:

   (i) $\log_2(x+3)^2 = \log_2(x^2+9)$.   **[1 mark]**

   (ii)   If $\log_2(x+4) = 2\log_2(x+1)$, then $\log_2(x+4) - \log_2(x+1) = 2$.   **[2 marks]**

   (iii) If $\log_a b = y$, then $y^a = b$.   **[2 marks]**

(b) Given that $\log_7 2 = 0.356$, and $\log_7 3 = 0.565$, find

    (i) $\log_7 \sqrt{3}$ **[3 marks]**

    (ii) $\log_7 14$ **[4 marks]**

(c) Solve the equation

$$\log_4(x^2 + 8x - 1) = 2 + \log_4(x - 1).$$

[8 marks]

♦♦♦♦♦♦♦   *END OF EXAM*   ♦♦♦♦♦♦