UNIVERSITY OF SWAZILAND

Final Examination 2006

Title of Paper : Elementary Quantitative Techniques

Program : B.A. Hums./B.A.S.S. I

Course Number : MS 001 (ii)

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of six (6) questions on FOUR (4) pages.
2. Answer ALL questions.
3. Non-programmable calculators may be used.

Special Requirements : None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

QUESTION 1

1 Given that $f(x) = x^3 - 6x^2 + 9x - 4$

1.1 Show that $x-1$ is a factor of $f(x)$. (2)

1.2 Find the y - intercept of $f(x)$ (2)

1.3 Find the x - intercept(s) of $f(x)$ (4)

1.4 Find the coordinates of the turning points of $f(x)$ using calculus. (6)

1.5 Hence or otherwise, sketch the graph of $f(x)$ (4)

1.6 Determine the equation of the tangent to the curve at the point (2; -2) (5)

23 MARKS

QUESTION 2

2.1 Use the definition of the derivative to evaluate the derivative of the following function:

$$f(x) = x^2 + 3$$

(7)

2.2 Use the rules of differentiation to evaluate the derivatives of the following functions:

2.2.1. $f(x) = x + 3$ (1)

2.2.2. $f(v) = 10v - \frac{v^2}{10}$ (2)

2.2.3. $f(y) = 3y^4 - 2y^3 + 7y^2 + 3y - 5$ (2)

2.2.4. $f(x) = (x-1)(x+6)$ (2)

2.2.5. $f(x) = \sqrt[3]{x} - \frac{3}{x}$ (LEAVE ANSWER WITH POSITIVE EXPONENTS) (3)

17 MARKS

QUESTION 3

3 A UNISWA student wanted to see how the volume of a gas changed when she played around with the pressure on the gas in a sealed container. The results that she got are shown in the table below.

PRESSURE (kPa)	VOLUME (cm^3)
1	20
5	4
8	2.5
10	2

3.1 Name the independent variable from the investigation. (1)

3.2 How many Pascals (Pa) are there in 1 kilopascal (kPa) (1)

3.3 Does this relationship reflect direct or inverse proportion? (1)

3.4 Hence or otherwise, calculate the volume of the gas when the applied pressure was 2 kPa. (3)

3.5 Draw a sketch graph of the graph that is likely to be obtained from this investigation if Volume was potted against pressure. Label your axes clearly. (2)

3.6 The results that the student got from her investigation fuelled her interest in trying other factors that she thought could affect the volume of the gas. She then tried out temperature. Here results are illustrated in the table below

Temperature (K)	VOLUME (cm^3)
273	22.4
292.8	24
451.4	37
?	49.2

The student recorded her temperature in another unit of temperature called the Kelvin (K).

$$Kelvins = Degrees\,Celsius + 273$$

For example 300K = 27°C, by using the formula above.

3.6.1. Name the type of proportion that is represented by the table above. (1)

3.6.2. Calculate the value of the temperature, in degrees Celsius (°C) that gave rise to a volume of 49.2cm^3 from the results shown in the table above. (3)

12 MARKS

QUESTION 4

4 Evaluate the following limits

4.1 $\lim_{x\to 3} \pi$ (1)

4.2 $\lim_{x\to -1} 5x+8$ (2)

4.3 $\lim_{t\to -1} \frac{t+1}{t^3-t}$ (3)

4.4 $\lim_{m\to 4} \frac{m^2-3m-4}{m-4}$ (3)

4.5 $\lim_{h\to 0} \frac{(1+h)^2-1}{h}$ (4)

13 MARKS

QUESTION 5

5 A ball is thrown vertically upwards. The height of the ball in metres above the ground, t seconds after it is thrown, is given by the formula $f(t) = 20t - 5t^2$.

5.1 Calculate the height of the ball after three seconds. (2)

5.2 After how many seconds does the ball reach maximum height? (3)

5.3 Determine the velocity (the *rate of change of the height of the ball*) of the ball after 1.5 seconds. (2)

5.4 After how many seconds is the velocity of the ball 12m/s? (3)

10 MARKS

QUESTION6 (25 MARKS)

6 Work out the following:

6.1 Evaluate the sum of the indicated series

$$\sum_{n=1}^{4} \frac{1}{n}$$

(2)

6.2 Find the sum of the first 20 terms in the series

$$1 + 3 + 5 + 7 + \ldots\ldots\ldots$$

(4)

6.3 Write down the first three terms in the following sequence

$$a_n = \frac{n}{n+1}$$

(2)

6.4 Write down the two missing numbers in the following sequence

$$2,\ -4,\ 8,\ -16,\ 32,\ -64,\ \ldots\ldots,\ \ldots\ldots\ldots$$

(2)

6.5 Evaluate the following integrals. Indicate integration constants where necessary.

6.5.1. $\int_1^3 4\ dx$ (3)

6.5.2. $\int x + 7\ \ dx$ (3)

6.5.3. $\int 2x\cos(x^2 + 3)dx$ (4)

6.6 Calculate the area of the shaded region below using calculus (5)